Frequency trees

Component Knowledge

- Complete a frequency tree from given information.
- Calculate probabilities from a frequency tree

Key Vocabulary

Frequency	The number of times an event occurs.
Probability	The chance that something will happen.
Frequency tree	Used to record and organise the frequency of events occurring.

Frequency trees are a way of organising information. They can be used to solve probability problems.

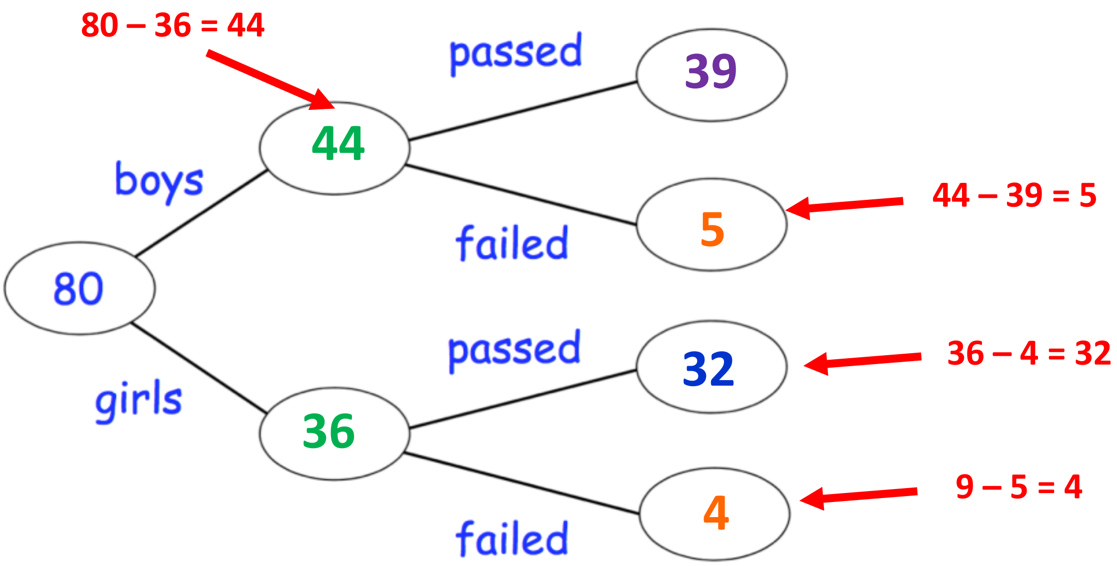
We start with the total number of items and then divide these items into two or more categories, writing down the frequency of items in each category.

A group of 80 boys and girls sat a test.

36 of the children are girls.

9 of the 80 children failed the test.

39 of the boys passed the test.



One of the boys is chosen at random.

Work out the probability that the boy failed the test. $\frac{5}{44}$

Number of boys who failed.

Total number of boys.

Online clip

U280

Frequency Tables

Component Knowledge

- Construct frequency tables.
- Read and interpret frequency tables.

Key Vocabulary

Frequency	The rate at which something occurs
Table	A logical way of displaying facts and figures
Tally	A way of displaying values using lines and dashes
Data	A collection of facts and figures
Discrete	Data that can only be set values e.g. you cannot have half of a person so counting people would be discrete data
Continuous	Data that can be measured and take any value e.g. height and time.

When we are dealing with a large amount of data, it is sometime impractical to display the data as a simple list. Frequency tables are a logical way of displaying large amounts of data which makes the data easier to analyse.

Frequency Table

Tally marks are used to help count things. Each vertical line represents one unit. The fifth tally mark goes down across the first four to make it easier to count. The frequency column is completed after all the data has been collected.

You must represent 5 like this.

Eye Colour	Tally	Frequency
brown	𝍸 I	6
blue	𝍸 III	8
green	III	3
grey	IIII	4
hazel	𝍸	5

Grouped Frequency Tables

When we have a large range of values it is better to group the data so the table is easier to read.

Note: You must ensure there is no overlap in the groupings.

20 students took a science test.
Place the data shown below in the grouped frequency table.
What is the modal class for the data?

25	32	31	52	45
27	55	28	42	44
46	23	51	48	26
20	51	49	33	41

Marks, *m*	Tally	Total
20-29	𝍸 I	6
30-39	III	3
40-49	𝍸 II	7
50-59	IIII	4

The values with the highest frequency show the modal class for the data. E.g. The modal class is 40-49.

Online clips

M945, M899, M441

Probability

Component Knowledge

- Understand what probability shows
- Understand probability notation
- Write a probability of a single event

Key Vocabulary

Probability	The mathematical chance, likelihood, of an outcome happening
Event	The “thing” that is being completed/done/observed/counted
(Event) Outcome	What happens when the event is performed
Probability scale	A numerical scale from 0 to 1, with 0 being an impossible outcome and 1 being an outcome certain to happen
Mutually exclusive (event) outcomes	When outcomes cannot happen at the same time eg being an adult and being a child, you **cannot be both**
Exhaustive (event) outcomes	When a set of outcome cover all possibility with no gaps eg it snowing and it not raining

Probability:

The probability of an (event) outcome A, happening is

$$P(outcome\ A) = \frac{number\ of\ ways\ outcome\ A\ can\ happen}{number\ of\ ways\ any\ outcome\ can\ happen}$$

e.g. the probability of rolling a number 4 on a regular 6 sided dice

$Outcome\ "4": \mathbf{4},\ so\ \ \mathbf{1\ option}$

$$P(roll\ a\ 4) = \frac{1}{6}$$

$All\ possible\ outcomes: \mathbf{1, 2, 3, 4, 5}\ or\ \mathbf{6},\ so\ \ \mathbf{6\ possibilities\ altogther}$

e.g. the probability of rolling a number greater than 4 on a regular 6 sided dice

$Outcomes\ "greater\ than\ 4": \mathbf{5}\ or\ \mathbf{6},\ so\ \ \mathbf{2\ options}$

$$P(roll\ a\ number\ greater\ than\ 4) = \frac{2}{6}$$

$All\ possible\ outcomes: \mathbf{1, 2, 3, 4, 5}\ or\ \mathbf{6},\ so\ \ \mathbf{6\ possibilities\ altogther}$

Online clips

M655, M941, M938, M755

Tree diagrams – independent

Component Knowledge

- Fill in missing values on a tree diagram
- Complete a tree diagram
- Find probabilities from a tree diagram

Key Vocabulary

Independent	An event that is not affected by other events
Probability	The chance that something happens
Event	One (or more) outcomes of an experiment
Outcome	A possible result of an experiment
Tree diagram	A diagram of lines connecting nodes, with paths that go outwards and do not loop back

Key Concepts

Independent events are events which do not affect one another.

Eg – replacing a counter before taking another from a bag

Probabilities on each set on branches add up to 1.

Probabilities can be written as fractions or decimals.

Probability Rules

The AND rule for probability states that the probability of A and B is the probability of A x the probability of B

The OR rule for probability states that the probability of A or B is the probability of A + the probability of B

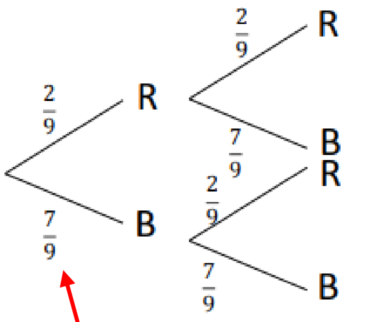
Example

There are red and blue counters in a bag.

The probability that a red counter is chosen is 2/9.

A counter is chosen and replaced, then a second counter is chosen.

Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.



Prob of two reds:

$$\frac{2}{9}\times\frac{2}{9}=\frac{4}{81}$$

Prob of two blues :

$$\frac{7}{9}\times\frac{7}{9}=\frac{49}{81}$$

Prob of same colours:

$$\frac{4}{81}+\frac{49}{81}=\frac{53}{81}$$

Note – the probability of a blue counter is found by doing 1 – 2/9 to give 7/9

Online clips

U558

Tree diagrams - dependent

Component Knowledge

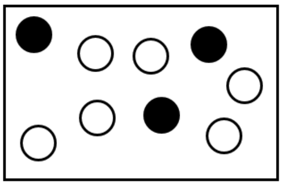
- Draw a probability tree for dependent events
- Calculate probabilities from a dependent event tree diagram

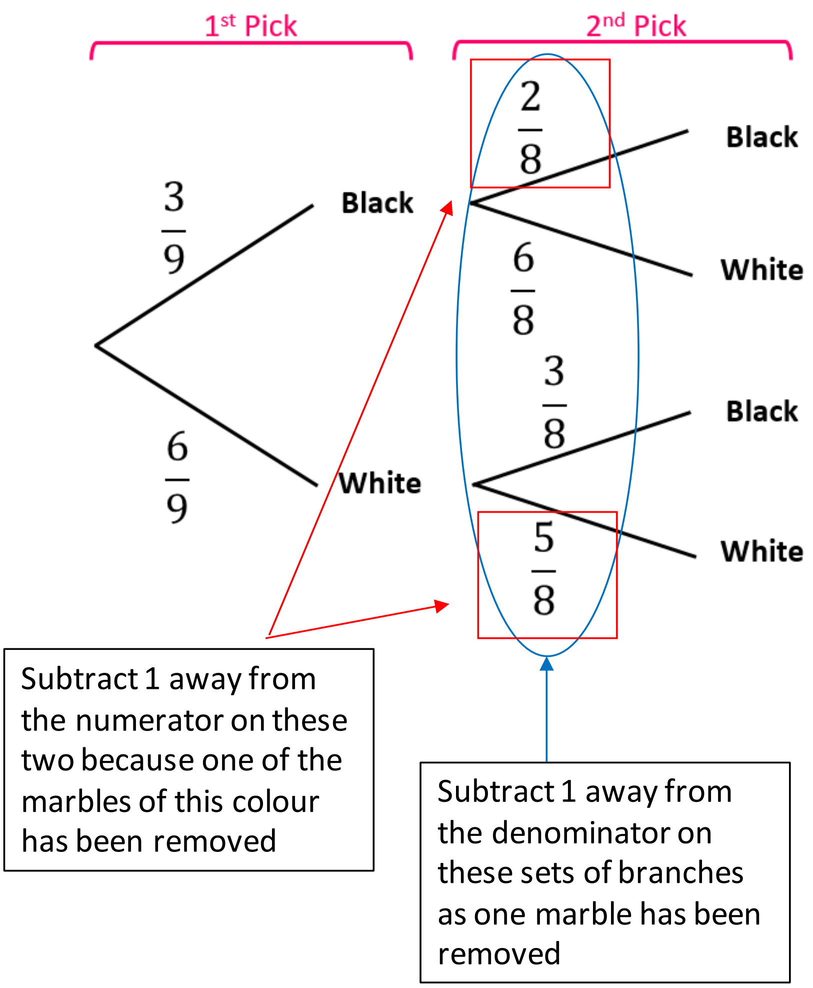
Key Vocabulary

Probability	The chance that something will happen
Event	The outcome of a probability
Tree diagram	Tree diagrams show all the possible outcomes of an event and helps to calculate their probabilities. Each set of branches must add up to 1.
Dependent	The outcome of a previous event does influence/affect the outcome of a second event.
Outcome	The result of a single performance of an experiment
AND rule	The outcome has to satisfy both conditions at the same time. Multiply the probabilities together.
OR rule	The outcome has to satisfy one condition, or the other, or both. Add the probabilities together.

Dependent tree diagrams

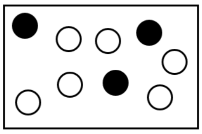
There are black and white marbles in the box. One is picked – **and not replaced** – then another is picked. Draw a probability tree to show this information.



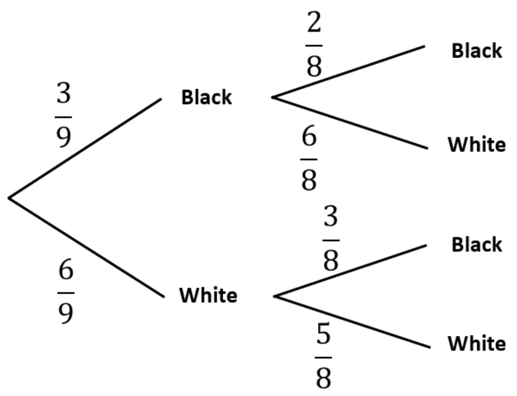


Dependent tree diagrams – calculating probabilities

There are black and white marbles in the box. One is picked – **and not replaced** – then another is picked. Draw a probability tree to show this information.



1st Pick | 2nd Pick | Result



BB $\frac{3}{9} \times \frac{2}{8} = \frac{6}{72}$

BW $\frac{3}{9} \times \frac{6}{8} = \frac{18}{72}$

WB $\frac{6}{9} \times \frac{3}{8} = \frac{18}{72}$

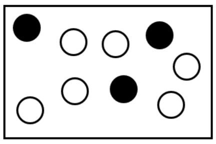
WW $\frac{6}{9} \times \frac{5}{8} = \frac{30}{72}$

Look for the results where the marbles are the same. In this example it is BB and WW. Add the probabilities together to get the answer.

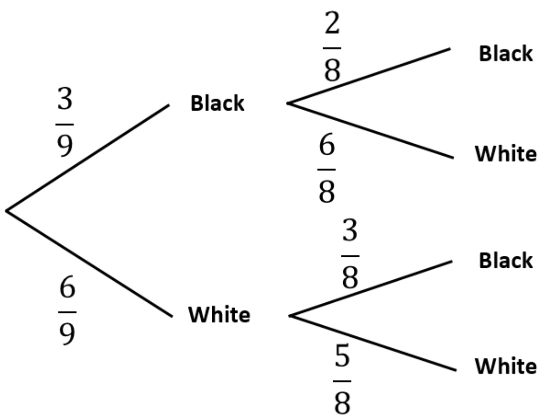
What is the probability two marbles of the **same colour** are picked?

P(Same colour) = $\frac{6}{72} + \frac{30}{72} = \frac{36}{72}$

There are black and white marbles in the box. One is picked – **and not replaced** – then another is picked. Draw a probability tree to show this information.



1st Pick | 2nd Pick | Result



BB $\frac{3}{9} \times \frac{2}{8} = \frac{6}{72}$

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WB $\frac{6}{9} \times \frac{3}{8} = \frac{18}{72}$

WW $\frac{6}{9} \times \frac{5}{8} = \frac{30}{72}$

Look for the results where at least one marble is B. In this example it is BB, BW and WB. Add the probabilities together to get the answer.

What is the probability **one or more** black marbles are picked?

P(1+ Black) = $\frac{6}{72} + \frac{18}{72} + \frac{18}{72} = \frac{42}{72}$

Online clips

U729

Two-way Tables

Component Knowledge

- Construct two-way tables.
- Read and interpret two-way tables.
- Find probabilities using two-way tables.

Key Vocabulary

Two-way table	A table which shows two variables at the same time- we can read them vertically and horizontally.
Horizontal	Reading from left to right or right to left
Vertical	Reading the table top to bottom or bottom to top
Variable	A way of organising data according to a shared characteristic e.g eye colour, age

We use two-way tables to compare 2 variables

To construct a two-way table, we need two variables. One variable is featured as the top row within the two-way table (read horizontally), and the other variable features on the first column of the table (read vertically).

Example This two way table shows a data set about what students eat for lunch.

	Boys	Girls	Total
Cooked food	18	22	40
Packed lunch	17	33	50
Total	35	55	90

The first column shows the type of food chosen.

The top row shows boy or girl.

17 boys had a packed lunch

90 students were asked in total (40+50=90 and 35+55=90

Example: 80 children went on a school trip.

They went to London or to York.

23 boys and 19 girls went to London.

14 boys went to York.

(a) Use this information to complete the two-way table.

	London	York	Total
Boys	23	14	
Girls	19		
Total			80

Step 1- fill in all known values from the question.

Total = 80

Boys in London = 23

Girls in London = 19

Boys in York = 14

Example: 80 children went on a school trip.

They went to London or to York.

23 boys and 19 girls went to London.

14 boys went to York.

(a) Use this information to complete the two-way table.

Step 2- calculate missing values using the known values. Remember both the horizontal and vertical totals must equal the overall total, in the case below, = 80.

	London	York	Total
Boys	23	14	37
Girls	19	24	43
Total	42	38	80

23 + 19 = 42

Boys total

80 – 37 = 43

Girls total

23 + 19 = 42

London total

80 – 42 =38

York total

38 – 14 = 24

Girls in York

Interpreting two-way tables

We can now use the fully completed two-way table to interpret the data.

	London	York	Total
Boys	23	14	37
Girls	19	24	43
Total	42	38	80

Questions could look like this:

a) How many students went to London?

We can read from the table vertically and see there **were 42 students who visited**

b) One of these 80 students is chosen at random.
What is the probability that this student visited London?

We can read from the table vertically and see there **were 42 students who visited London.**

So, the $P(a\ student\ visits\ London) = \frac{42}{80}$

c) A student is picked at random.
Given they are a girl, what is the probability they went to York?

We can read the table to find the **total girls = 43** and the **girls who visited York = 24**

So, the $P(given\ the\ student\ is\ a\ girl,\ they\ visit\ York) = \frac{24}{43}$

Online clip M899

Estimation

Component Knowledge

- Estimate values of numeric problems
- Estimate values of worded problem solving questions
- Identify whether an estimation is an under-estimate or an over-estimate

Key Vocabulary

Round	Making a number simpler whilst keeping its value close to the original.
Significant figures	The number of digits in a value that carry a meaning to the size of the number.
Estimate	Find a value that is close to the right answer by rounding.

When estimating any calculation, you need to round every number to one significant figure

Estimating Calculations

Estimate 39 x 4.85

$$39 \times 4.85$$

$$40 \times 5$$

$$= 200$$

Estimate

$$\frac{52 \times 6.78}{0.51}$$

First round all the numbers to 1 significant figure

$$\frac{50 \times 7}{0.5}$$

Then calculate the numerator

$$\frac{350}{0.5}$$

Dividing by 0.5 is the same as multiplying by 2

$$750$$

Significant figures

Example

Round 3786 to one significant figure

Th	H	T	U
3	7	8	6

The first significant figure is in the thousands column so to the nearest thousand it is 4000

Estimation worded problems

Mr Sykes wants to buy a calculator for every student in year 11. There are 105 students in year 11. Each calculator costs £6.99

Work out an estimate for the amount of money Mr Sykes will spend on calculators.

First round all the numbers to 1 significant figure

$105\ students$ → $100\ students$

$£6.15$ → $£6$

$$100 \times £6 = £600$$

Online clips

M994, M131,M878

How to decide if your solution is an underestimate or overestimate.

Decide if you have made each number bigger or smaller by rounding. When dividing remember that if you divide by a number that has been rounded up, your answer will be an underestimate and vice versa

For example: In the calculator example above we rounded the cost and number of students down so this is an under estimate of the cost.

Error Intervals

Component Knowledge

- To use understand how to round to different degrees of accuracy.
- To be able to write error intervals when rounding using correct inequality notation.
- To be able to write error intervals when rounding using correct inequality notation.

Key Vocabulary

Rounding	Rounding means making a number simpler but keeping its value close to what it was. The result is less accurate, but easier to use.
Accuracy	How close the rounded value is to the original value.
Place value	The value of the digit in a number
Lower bound	The smallest possible value that can be rounded to the number given.
Upper bound	The largest possible value the rounded value can take.
Truncation	Truncation comes from the word truncare, meaning "to shorten". The number is cut off at a certain point.
Inequality notation	Symbols used to describe the relationship between two expressions that are not equal to one another.

Inequality Notation

All error intervals look the same like this:

______ $\leq n <$ ______

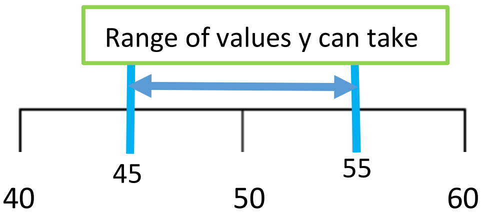
The value, n, can be greater or equal to this number.

The value, n, can only be less than this number but we use it to make any calculations easier to perform, should we need to.

Error intervals- rounding according to place value

Example 1- Frank rounds a number, y, to the nearest ten. His result is 50 Write down the error interval for y.

Begin by finding the ten, in this case, greater than and less than 50.



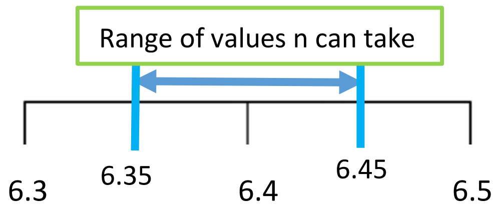
The midpoint between 40 and 50 is 45. This is the lower bound.

The midpoint between 50 and 60 is 55. This the upper bound (this can never = 55 but can be as large as 54.9999999….. 55 is easier to calculate with. Additionally, we use < as well.

The answer is $45 \leq y < 55.$

Example 2- Freya rounds a number, n, to one decimal place. Her result is 6.4 Write down the error interval for n.

Begin by finding the tenth, in this case, greater than and less than 6.4. **(Note: 1dp = tenths column.)**



The midpoint between 6.3 and 6.4 is 6.35. This is the lower bound.

The midpoint between 6.4 and 6.5 is 6.45. This the upper bound (this can never = 6.45 but can be as large as 6.49999999….. 6.45 is easier to calculate with. Additionally, we use < as well.

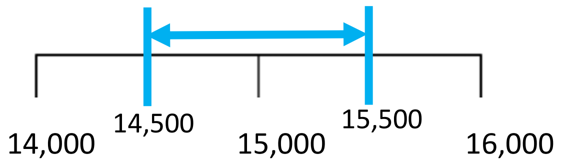
The answer is $6.35 \leq n < 6.45.$

Error intervals- rounding according to significant figures

Depending on the size of the number, the rounding will change when rounding to significant figures. Rounding like this keeps all numbers rounded to the same degree of accuracy relative to the size of the number.

Example 3- A number, g, is 15,000 when rounded to 2 significant figures. Write down the error interval.

Begin by finding the place value of the 2nd significant figure, in this case, this is 5000. This means we are rounding to 2 sig figs = rounding to nearest thousand.



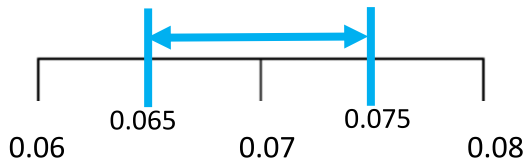
The midpoint between 14,000 and 15,000 is 14500. This is the lower bound.

The midpoint between 15,000 and 16,000 is 15,500. This the upper bound.

The answer is $14{,}500 \leq g < 15{,}500.$

Example 4- A number, x, is 0.07 when rounded to 1 significant figure. Write down the error interval.

Begin by finding the place value of the 1st significant figure, in this case, this is 0.07. This means we are rounding to 1 sig fig =rounding to nearest hundredth.



The midpoint between 0.06 and 0.07 is 0.065. This is the lower bound.

The midpoint between 0.07 and 0.08 is 0.075. This the upper bound.

The answer is $0.065 \leq x < 0.075.$

Error intervals- truncation

Be careful when reading error interval questions as truncating is not rounding like place value. The number has been "chopped", which means the value given **IS THE LOWER BOUND.** It commonly applies to decimals.

Example 5- State the error interval of 4.5 when it has been truncated to 1 decimal place.

Begin by finding the tenth, in this case, greater than 4.5. (**Note: 1dp = tenths column.**) This is the upper bound.

Remember: the value cannot equal 4.6!

The answer is $4.5 \leq n < 4.6.$

Online clip

M730

Inequalities

Component Knowledge

- Understand and use inequality notation
- Represent the solution set of an inequality on a number line
- Decide whether a number satisfies an inequality
- Form an inequality from a question and solve it

Key Vocabulary

Inequality	An inequality shows that two quantities are (may) not be equal
Less than	This is shown by the symbol $<$
Less than or equal to	This is shown by the symbol $\leq$
Greater than	This is shown by the symbol $>$
Greater than or equal to	This is shown by the symbol $\geq$
Integer	A whole number

Notation

$x > 2$ means x **is greater than 2**

$x < 3$ means x **is less than 3**

$x \geq 1$ means x **is greater than or equal to 1**

$x \leq 6$ means x **is less than or equal to 6**

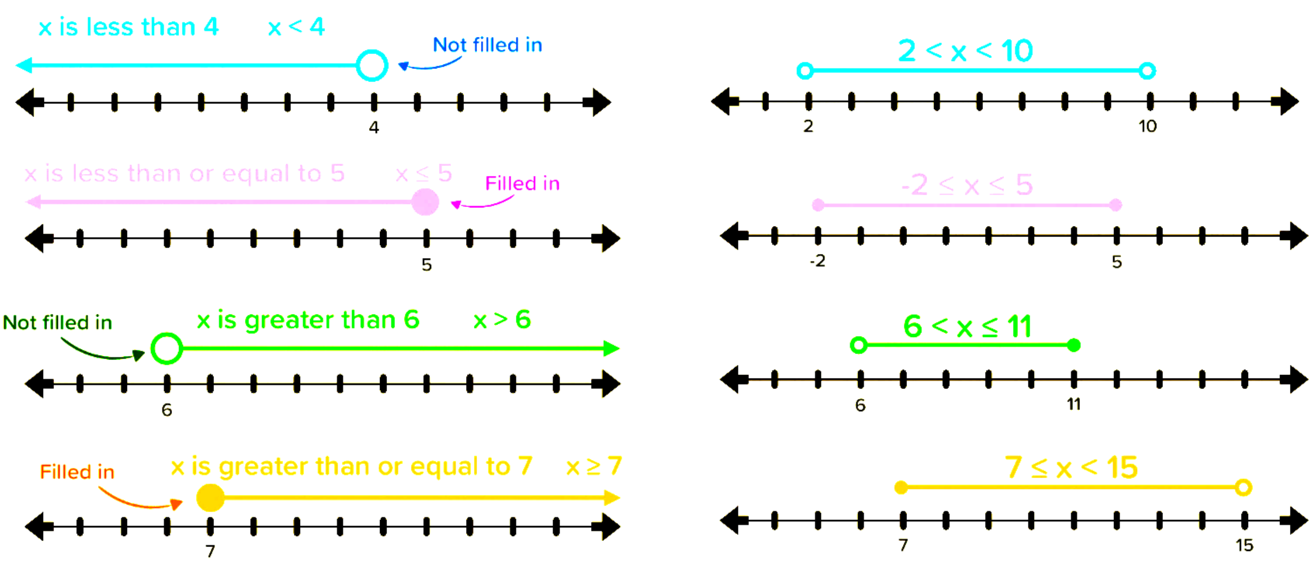
Examples:

$x \geq 1$ is **true** for $x = 6, 2.5$ and 1

$x < 5$ is **false** for $x = 10, 5.05$ and 5

The set of *integers* which **satisfy** $-2 \leq x < 3$ is $\{-2, -1, 0, 1, 2\}$

The set of numbers *satisfying* an inequality can be *represented* on a number line:



Inequalities can be **solved** by the same method as used for equations:

a) $x - 7 \le 12$

b) $5y > 40$

c) $\frac{b}{3} \ge -2$

One-step solution

$$\begin{array}{ccc} & x - 7 \le 12 & \\ +7 & & +7 \\ & x \le 19 & \end{array}$$

$$\begin{array}{ccc} & 5y > 40 & \\ \div 5 & & \div 5 \\ & y > 8 & \end{array}$$

$$\begin{array}{ccc} & \frac{b}{3} \ge -2 & \\ \times 3 & & \times 3 \\ & b \ge -6 & \end{array}$$

Inverse operation

a) $5(x - 1) < 3.5$

$\frac{b}{6} + 2 \ge 1$

Two-step solution

$$\begin{array}{ccc} & 5(x - 1) < 3.5 & \\ \div 5 & & \div 5 \\ & x - 1 < 0.7 & \\ +1 & & +1 \\ & x < 1.7 & \end{array}$$

$$\begin{array}{ccc} & \frac{b}{6} + 2 \ge 1 & \\ -2 & & -2 \\ & \frac{b}{6} \ge -1 & \\ & & \\ & b \ge -6 & \end{array}$$

Make sure you write an inequality symbol

<u>Online clips</u>

M384, M118