Trigonometry

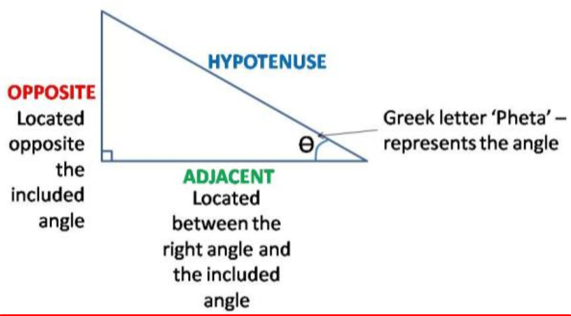
Component Knowledge

- Recall the three trigonometric ratios
- Correctly label the sides of a right-angled triangle with Opp, Adj, Hyp
- Identify the correct trigonometric ratios to use.
- Use the correct trigonometric ratio to find the missing side or angle.
- To identify exact values for key angles using trigonometric ratios.

Key Vocabulary

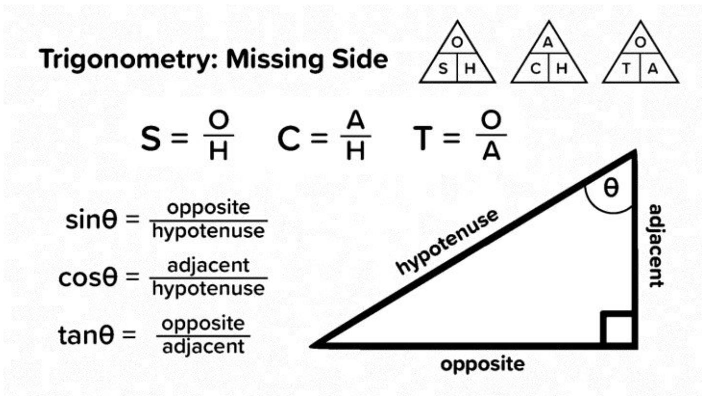
Trigonometry (trig)	Trigonometry helps us find angles and lengths in right-angled triangles.
Trigonometric ratios	There are three trigonometric ratios, depending on the position of the unknown sides and angles.
Hypotenuse side (Hyp)	The length of the longest side of a right-angled triangle.
Opposite side (Opp)	The length of the side opposite the given angle.
Adjacent side (Adj)	The length of the side next to the given angle and right-angle.
Sine ratio (sin)	Is used to describe the relationship between the given angle, Opposite side and Hypotenuse
Cosine ratio (cos)	Is used to describe the relationship between the given angle, Adjacent side and Hypotenuse
Tangent ratio (tan)	Ratio used to describe the relationship between the given angle, Opposite side and Adjacent side.

Labelling the sides



Hypotenuse will **always** be opposite the 90° angle. The Opposite and Adjacent will change depending on where the given angle (Θ) is on the diagram.

Trig ratios



Finding the missing side

- numerator

Remember to show your full calculator answer too, should you have a decimal, and then round to get your final answer.

To find the length of b

Step 1: Label each side
Step 2: Choose the correct formula
Since we know the hypotenuse and need to find the adjacent side, we can use cos
Step 3: Substitute the values into the formula

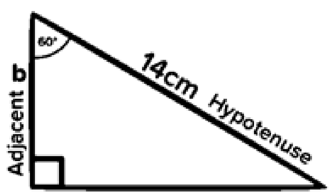
$\cos(60) = \frac{b}{14}$

Step 4: Rearrange to find b

$b = \cos(60) \times 14$

b = 7

SOH CAH TOA



Finding the missing side- denominator

Q Find the length of h

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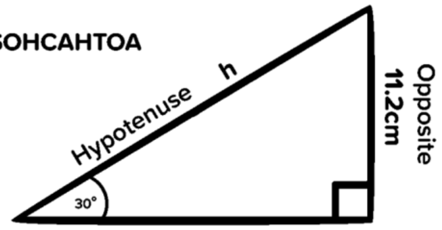
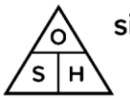


Step 1 Label each side

Step 2 Choose the correct formula

Since we know the opposite side and need to find the hypotenuse, we can use sin



$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

Step 3 Substitute the values into the formula

$\sin(30) = \frac{11.2}{h}$

Step 4 Rearrange to find h

$h = \frac{11.2}{\sin(30)}$

A 22.4cm

Here we have multiplied both sides by h and then divided both sides by sin (30)

Finding the missing side- angle

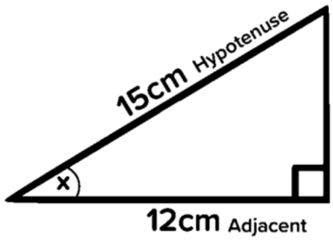
To type $\sin^{-1}$ into your calc, press SHIFT and then the SIN button.

To type $\cos^{-1}$ into your calc, press SHIFT and then the COS button.

To type $\tan^{-1}$ into your calc, press SHIFT and then the TAN button.

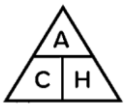
Q Calculate the value of the missing angle x

Give your answer to 1 decimal place.



Step 1 Label the triangle

Step 2 Choose the correct formula



$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

Step 3 Substitute and solve

$\cos(x) = \frac{12}{15}$

$x = \cos^{-1}\left(\frac{12}{15}\right)$

$= 36.8698...$

1 decimal place

A 36.9°

Exact trigonometric values- LEARN BY HEART!!

	0°	30°	45°	60°	90°
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$undefined$

Online clips

U605, U283, U545, U627

Best Buys

Component Knowledge

- Compare products with different quantities and values.

Key Vocabulary

Best Buy	Comparing products to see which is the best value for money
Value	A measure of somethings worth in money
Multiple	A number that may be divided into another number a certain amount of times with no remainder
Direct Proportion	When one value is increased or decreased a related value will increase or decrease at a constant rate
Unitary	Relating something to a unit or units
Method	A particular procedure for accomplishing or approaching something
Deal	An agreement made when trading goods or services

Best Buys

Best buys problems involve assessing which item is the best value for money.

In order to compare deals:

1. **Note the cost of the items and the number of items for each deal.**
2. **Calculate the price for an equivalent number of items for each deal.** For the unitary method, this is the price of a single item. For the common multiples method, this is the price of a common number of items.
3. **Compare the prices of the equivalent quantities.**

The Unitary Method

Boxes of tissues come in 3 sizes:

A) 24 Tissues = £7.99
B) 20 Tissues = £7.33
C) 15 Tissues = £5.65

Which box is the best value for money?

Step 1 – Find the price per tissue for each box

Box A = £7.99 ÷ 24
= £0.333 or 33.3p/tissue

Box B = £7.33 ÷ 20
= £0.367 or 36.7p/tissue

Box C = £5.65 ÷ 15
= £0.377 or 37.7p/tissue

Questions like this can typically be solved by first finding the amount per unit (e.g. weight per cm; price per tissue). This is called the 'unitary method'

Step 2 – Identify which box is cheapest per tissue

The best value for money is Box A as it is the cheapest per tissue.

Online clip

M681

Finding a Comparative Amount

Sometimes it is better to compare reasonable amounts rather than using the unitary method.

Decide which product is the best value for money.

•Radox Handwash
500ml bottles on offer at 3 for 2
Price for each £1.90

300ml bottles on offer at buy one get one free
Price for each £1.88

3 x 500 = 1500ml
Cost 2 x £1.90 = £3.80

100ml cost £3.80 ÷ 15 = £0.25

2 x 300 = 600ml
Cost £1.88

100ml cost £1.88 ÷ 6 = £0.31

Pie charts

Component Knowledge

- Calculate angles in a pie chart
- Draw a pie chart from a table
- Interpret pie charts using fractions
- Interpret pie charts using angles

Key Vocabulary

Angle	The amount of turn between 2 lines.
Pie chart	A chart that displays data proportionally.
Protractor	Equipment used to measure and draw angles

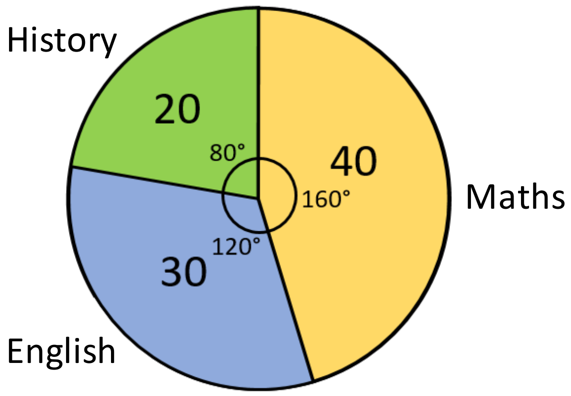
Drawing pie charts

How many degrees for one person? $\frac{360}{90} = 4°$

360 ÷ total = degrees for one person. In this example one person is 4°.

Subject	Number of Students	Calculation	Angle
Maths	40	40 × 4°	160°
English	30	30 × 4°	120°
History	20	20 × 4°	80°
Total	90		360°

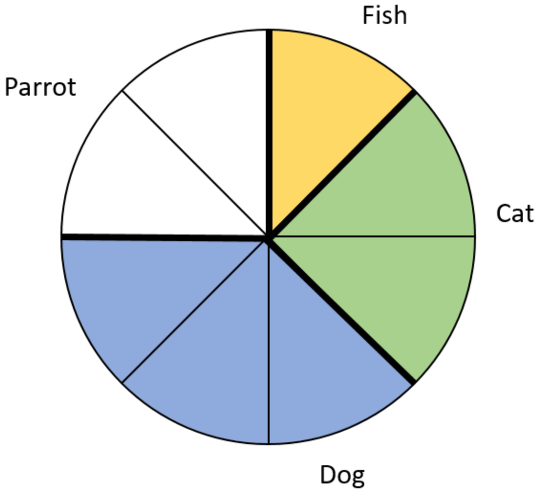
Multiply number of students by 4° to get the angle.



Draw the angles onto the pie chart. Label each part with what it is (subject in this example) and how many it represents (40 for Maths in this example).

Interpret pie charts (fractions)

A class of **32 students** were surveyed to find their **favourite pet**.
The **pie chart** shows the total answers. How popular was each animal?



The pie chart is split into 8 pieces, so each sector is worth $\frac{1}{8}$ of 32 = 4

Fish: $\frac{1}{8}$ of 32 = 4

Cat: $\frac{2}{8}$ of 32 = 8

Dog: $\frac{3}{8}$ of 32 = 12

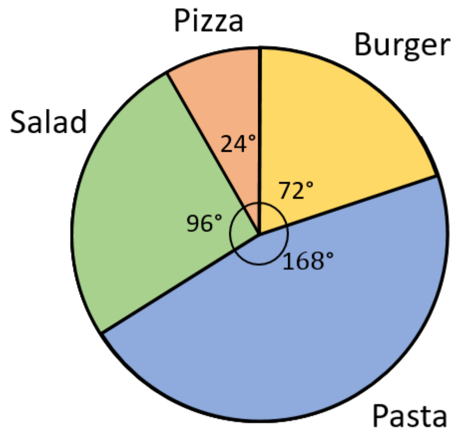
Parrot: $\frac{2}{8}$ of 32 = 8

Check that the totals add up to the original total in the question.
(4 + 8 + 12 + 8 = 32)

Interpret pie charts (angles)

150 students were surveyed about their favourite food.

Favourite Food	Angle	Calculation	Frequency
Burger	72°	$\frac{72}{360} \times 150$	30
Pasta	168°	$\frac{168}{360} \times 150$	70
Salad	96°	$\frac{96}{360} \times 150$	40
Pizza	24°	$\frac{24}{360} \times 150$	10



To calculate the frequency from a pie chart when you are given the angle, you do the opposite of what you do to calculate the angle.

Angle ÷ 360 × total frequency

Online clips

M574, M165

Index Laws

Component Knowledge

- To be able to apply the different index laws
- To be able to calculate negative indices
- To be able to calculate fractional indices

Key Vocabulary

Index notation	A way of writing numbers or letters that have been multiplied by themselves a number of times
Square number	The product of a number multiplied by itself
Cube number	The product of a number multiplied by itself three times.
Root	The inverse of a square number is a square root. The inverse of a cube number is a cube root
Reciprocal	1 divided by the number

Multiplication law

When multiplying the terms, we add the powers together

$$3^7 \times 3^5 = 3^{7+5} = 3^{12}$$

$$x^3 \times x^4 = x^{3+4} = x^7$$

The base number does not change

Division law

When dividing the terms, we subtract the powers.

$$2^7 \div 2^3 = 2^{7-3} = 2^4$$

Divides can only be written as fractions

$$\frac{5^{11}}{5^2} = 5^{11-2} = 5^9$$

$$\frac{y^5}{y^{-1}} = y^{5--1} = y^6$$

Subtracting a negative is the same as adding

Brackets law

$$(4^5)^3 = 4^{5\times 3} = 4^{15}$$

When raising to the power we multiply the powers together

$$(2x^4)^3 = 2^3 \times x^{4\times 3} = 8x^{12}$$

Facts

$$p = p^1$$

$$y^0 = 1$$

$$456^0 = 1$$

Anything to the power of zero is equal to 1

Index Laws – You can only use index laws when the base number is the same.

$$2^3 \times 4^5 \neq 8^{15}$$

Negative indices

A negative power performs the reciprocal

$$x^{-a} = \frac{1}{x^a}$$

Example

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Fractional

The denominator of a fractional power acts as a "root. The numerator of a fractional power acts as a normal power.

General rule

$$x^{\frac{a}{b}} = \left(\sqrt[b]{x}\right)^a$$

$$64^{\frac{2}{3}} = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$$

Changing the base

Write

$$(4)^3 \text{ as a power of } 2$$

$4 = 2^2$, so

$$(4)^3 = (2^2)^3 = 2^6$$

Example

Given that

$$3 \times \sqrt{27} = 3^n$$

Find the value of n

$$27 = 3^3$$

$$3 \times \sqrt{3^3}$$

$$3^1 \times (3^3)^{\frac{1}{2}}$$

A square root can be changed to the power of $\frac{1}{2}$

$$3^1 \times 3^{\frac{3}{2}} = 3^{1+\frac{3}{2}} = 3^{\frac{5}{2}}$$

Online clips

M135, M608, M150, M120 X647, X783

Factors, multiples & primes

Component Knowledge

- Identify factors and multiples
- Identify a prime number
- Complete a prime factor tree and write the number in index form
- Calculate HCF and LCM of 2 values using an appropriate method.

Key Vocabulary

Factor	Numbers that we can multiply together to get another number
Multiple	The result of multiplying a number by an integer
Prime	A number that only has two factors 1 and itself
Highest common factor	The greatest number that is a factor of 2 (or more) other numbers
Lowest common multiple	The smallest positive number that is a multiple of two or more numbers
Product	The answer when two or more values are multiplied together
Factorisation	Writing a number as a product of two or more smaller numbers
Integer	A whole number

Multiples: The result of multiplying a number by an integer. It is the times table of a number.

Multiples of 4: 4, 8, 12, 16,20 …
Multiples of 5: 5, 10, 15, 20, 25….

Multiples are the list of times tables

Factors: A number that divides exactly into another number without a remainder. It is often helpful to write them in pairs.

Write them in pairs first so you don't miss any!

20
1 x 20
2 x 10
4 x 5

Factors of 20 = 1, 2, 4, 5, 10, 20

HCF & LCM

Highest common factor

Find the HCF of 12 and 20

Factors of 12	Factors of 20
1 and 12	1 and 20
2 and 6	2 and 10
3 and 4	4 and 5

4 is the highest factor of both numbers

Lowest common multiple

Find the lowest common multiple of 4 & 6

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32 …
Multiples of 6: 6, 12, 18, 24, 30, 36 …

12 is the lowest number that appears in both times tables.

Prime Numbers

1	**2**	**3**	4	**5**	6	**7**	8	9	10
11	12	**13**	14	15	16	**17**	18	**19**	20
21	22	**23**	24	25	26	27	28	**29**	30
31	32	33	34	35	36	**37**	38	39	40
41	42	**43**	44	45	46	**47**	48	49	50
51	52	**53**	54	55	56	57	58	**59**	60
61	62	63	64	65	66	**67**	68	69	70
71	72	**73**	74	75	76	77	78	**79**	80
81	82	**83**	84	85	86	87	88	**89**	90
91	92	93	94	95	96	**97**	98	99	100

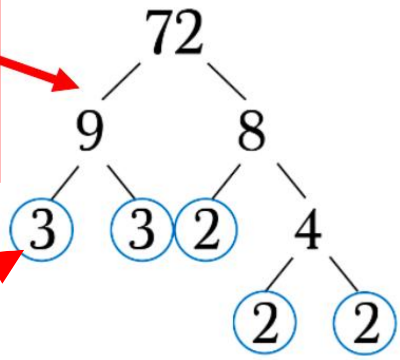
Prime numbers (highlighted)

Prime Factorisation

Write 72 as a product of its prime factors

We need to find pairs of numbers that multiply to give the number above.

When you get a prime number circle it.



$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$72 = 2^3 \times 3^2$$

If a number is repeated we write it as a power

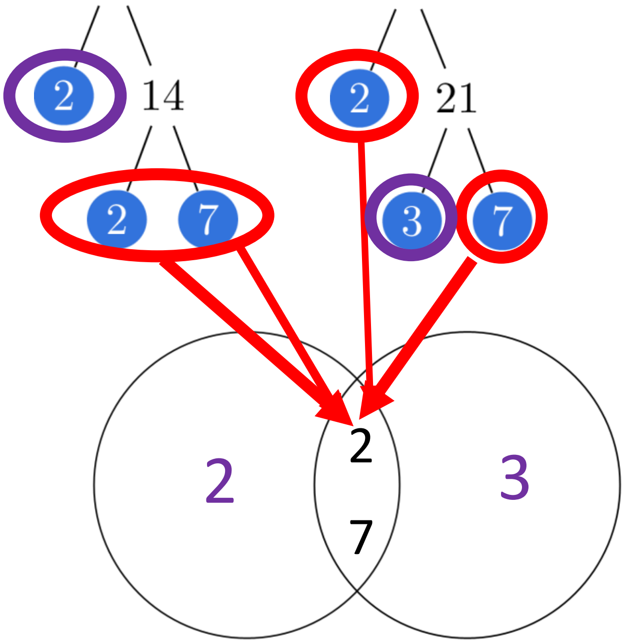
HCF & LCM using prime factors

Find the HCF and LCM of 28 and 42

First start by finding the Prime factors of 28 and 42

$28 = 2^2 \times 7$ $\quad$ $42 = 2 \times 3 \times 7$

Both trees have a 2 and 7 so those numbers go in the middle of the Venn diagram as they are shared.



The remaining numbers in the tree go in outside circles of the Venn

HCF – the highest common factor is found by multiplying the centre shared part of the Venn diagram

HCF = 2 x 7 = 14

LCM – the lowest common multiple is found by multiplying all the numbers in the Venn diagram

LCM = 2 x 2 x 3 x7 =84

Online clips

Speed, Distance & Time

Component Knowledge

- Calculate speed given distance and time (including fractional time).
- Use the correct formula to calculate speed, distance & time.

Key Vocabulary

Speed	A measure of how fast something is happening
Distance	A measure of how far it is from one place to another
Time	A measure of how long something takes to happen
Units	A quantity used as a standard measurement
Convert	To change something from one form to another
Average	A calculated central value of a set of numbers
Metric	A standard unit of measure using metres, kilograms and seconds
Imperial	A unit of measure developed in England. E.g. miles, pounds, gallons etc

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Distance} = \text{Speed} \times \text{Time}$$

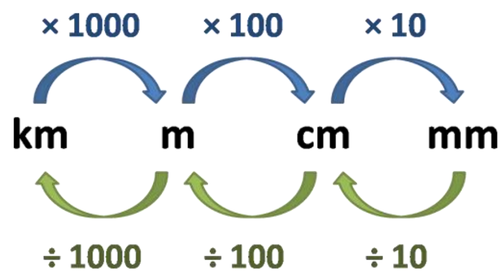
$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Units of speed include: m/s (metres per second), mph (miles per hour), Km/h (kilometres per hour).

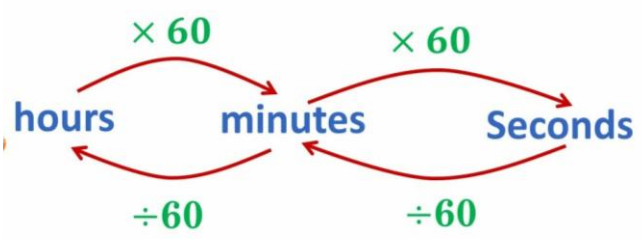
Units of distance include: m (metres), km (kilometres) miles.

Units of time include: s (seconds), min (minutes), h (hours).

Useful conversions



5 miles ≈ 8 kilometres



Example 1

Jim travels 45 miles in 3 hours.

What was his average speed in mph?

$$Speed = \frac{Distance}{Time} = \frac{45}{3}$$

$$\underline{= 15mph}$$

Example 2

Jess travels 45 miles in 1 hour 30 mins.

What was her average speed in mph?

1 hour 30 mins
$= 1\frac{30}{60}$ h
= 1.5 hours

$$Speed = \frac{Distance}{Time} = \frac{45}{1.5}$$

$$\underline{= 30mph}$$

Example 3

Jim drives at 40 mph for 3 hours.

How far did he travel?

$$Distance = Speed \times Time$$

$$Distance = 40 \times 3$$

= 120 miles

Example 4

For 15 minutes Sally ran at

an average speed of 20 km/h

$$Distance = Speed \times Time$$

$$Distance = 20 \times 0.25$$

= 5 km

Note: there are different units of time so we convert mins to hours.

15 mins = $\frac{15}{60}$ = 025 h

Example 5

A train travels 300 miles at 60 mph.

How long did this take?

$$Time = \frac{Distance}{Speed} = \frac{300}{60}$$

$$= 5\ hours$$

Example 5

A runner travels 3 km at 5 m/s.

How long did they take?

$$Time = \frac{Distance}{Speed} = \frac{3000}{5}$$

Note: there are different units of distance so we convert km to m.

3km = 3000m

$$= 600 seconds$$

$$= 10\ minutes$$

Note: this is not a sensible unit. We convert 600s to mins.

600s = 10 mins

Multi-Part Journeys

Julie drove 45km from Bath to Bristol.

She then drove 68km from Bristol to Cardiff.

Julie's average speed from Bath to Bristol was 50km/h

Julie took 105 minutes to drive from Bristol to Cardiff.

Work out Julie's average speed for her total drive from Bath to Cardiff.

Creating a table can help solve problems with multi-part journeys.

	Speed	Distance	Time
Bath to Bristol	50 km/h	45 km	0.9 h
Bristol to Cardiff	We do not need this	68 km	105 mins (1.75 h)
Total	**42.6 km/h**	113 km	2.65 h

We cannot just find the second speed and take the mean of the 2 values because the distances are different.

Use the formula to find the missing value so we can find the total distance and total time by adding them (we must convert the time to a decimal).

$$Average\ Speed = \frac{Total\ Distance}{Total\ Time} = \frac{113}{2.65} = 42.6 km/h (1.d.p)$$

Online clips

U151, M515

Pressure

Component Knowledge

- Calculate the pressure exerted on an object using the formula.
- Calculate the force exerted by an object using pressure and area.
- Calculate the area using pressure and force.

Key Vocabulary

Pressure	The effect of a force over an area.
Force	Force is push or pull. Measures in Newtons (N).
Area	The amount of space taken up on a flat surface.
Gravity	The force that attracts a body towards any other physical body that has mass.
Measure	To find a number that shows the size or amount of something.

Key Concepts

Whenever an object rests on a solid surface, the surface pushes back against the object, balancing the weight.

The effect that the force of gravity has on the surface depends on the size of the force and the area it is acting over. This effect is called pressure.

Pressure can be increased by increasing the size of the force or decreasing the area.

Formulae

$$Pressure = \frac{Force}{Area}$$

$$Area = \frac{Force}{Pressure}$$

$$Force = Pressure \times Area$$

Examples

A tracked excavator has a weight of 210,000N. The area in contact with the ground is $4m^2$.

$$Pressure = \frac{Force}{Area} = \frac{210{,}000N}{4m^2} = 52{,}500\ N/m^2$$

A man weighs 880N and his shoes have an area of $500cm^2$. What pressure does he put on the floor?

$$Pressure = \frac{Force}{Area} = \frac{800N}{500cm^2} = 1.6\ N/cm^2$$

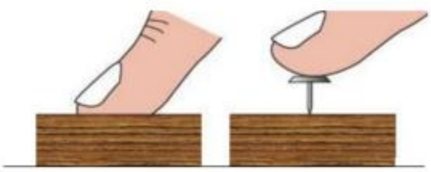
Units

Force is typically measures in Newton's (N)

Sometimes pressure is measures in Pascals (Pa)

- 1 Pa is the same as 1 N/m^2
- 1000 Pa equals 1 kilopascal (kPa)

Visual Representation



The drawing pin will sink into the wood as it has a small surface area which **concentrates** the force.

The finder won't sink in as it has a large surface area which **spreads out** the force.

Online clips

U527, U842

Density, mass and volume

Component Knowledge

- Calculate simple density, mass or volume
- Calculate more complex density, mass or volume
- Combining mass and volume to find density of a compound.

Key Vocabulary

Density	A measure of how tightly the mass of an object is packed into the space it takes up. If an object is heavy and small it will have a higher density
Mass	The mass of an object is the quantity of matter it contains. It never changes.
Volume	Volume is defined as the space occupied within the boundaries of an object in three-dimensional space
Units	The unit of measure used to describe density, mass and volume.
Compound measurement	A measure made up of two or more measurements (e.g. speed, pressure, density)

Formulae for density, mass and volume

$$Density = \frac{Mass}{Volume}$$

$$Volume = \frac{Mass}{Density}$$

$$Mass = Density \times Volume$$

Calculate density

A solid silver spoon has a mass of 65.1g. The volume of the spoon is 6.2cm^3. Calculate the density of silver.

Density = $\frac{Mass}{Volume}$ ← Write out the formula

Density = $\frac{65.1g}{6.2\ cm^3}$ ← Substitute in the values from the question

Density = 10.5 g/cm^3 ← Remember to include the units in the final answer

Calculate volume

Iron has a density of 7.8g/cm^3. A solid iron statue has a mass of 877.5g. Work out the volume of the statue.

Volume = $\frac{Mass}{Density}$ ← Write out the formula

Volume = $\frac{877.5g}{7.8\ g/cm^3}$ ← Substitute in the values from the question

Volume = 112.5 cm^3 ← Remember to include the units in the final answer

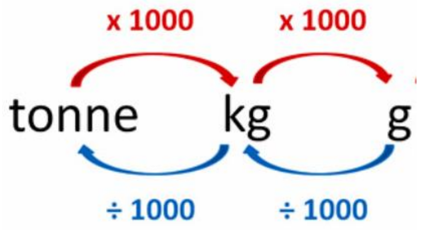
Calculate mass

A piece of plastic has a density of 1.3g/cm^3 and a volume of 100cm^3. Work out the mass of the piece of plastic.

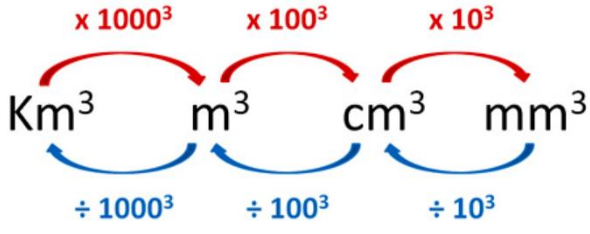
$Mass = Density \times Volume$ ← Write out the formula

$Mass = 1.3g/cm^3 \times 100cm^3$ ← Substitute in the values from the question

$Mass = 130g$ ← Remember to include the units in the final answer



Useful Conversions



Calculate more complex density, mass or volume

When calculating more complex density, mass or volume you may need to do a calculation before you can then substitute the values from the question into the formula. You made need to calculate the volume of the object first or you may need to change the units of mass or volume so that they are the same.

A glass cube of side length 5cm has a mass of 306.25g. Calculate the density of the glass.

$5 \times 5 \times 5 = 125cm^3$ ← Calculate the volume of the cube

Density = $\frac{Mass}{Volume}$ ← Write out the formula

Density = $\frac{306.25g}{125cm^3}$ ← Substitute in the values you know

Density = 2.45g/cm³ ← Remember to include the units in the final answer

A garden ornament has a volume of 0.05m³. The ornament is made from a stone that has a density of 6.4g/cm³. Calculate the mass of the ornament. Include suitable units.

$0.05m^3 \times 1{,}000{,}000 = 50{,}000cm^3$ ← Units need to be the same. Convert m³ into cm³

$Mass = Density \times Volume$ ← Write out the formula

$Mass = 6.4g/cm^3 \times 50{,}000cm^3$ ← Substitute in the values from the question

$Mass = 320{,}000g$ ← Remember to include the units in the final answer

$Mass = 320kg$ ← Change the units to kg as it is more suitable than g

Combining mass and volume to find new density

When combining mass and volume to find a new combined density you cannot just add the two densities together. You have to find the total mass and the total volume of the new substance and then use these amounts to calculate the density of the compound (Sterling silver in the example below).

Some sterling silver is made with 900 g of silver and 90 g of copper. The density of silver is 10 g/cm³. The density of copper is 9 g/cm³. What is the density of the sterling silver?

a)

	Silver	Copper	Sterling silver
Density	**10 g/cm³**	**9 g/cm³**	
Mass	**900g**	**90 g**	
Volume			

← Fill in what you know from the question

b)

	Silver	Copper	Sterling silver
Density	10 g/cm³	9 g/cm³	
Mass	900g	90 g	
Volume	**90cm³**	**10 cm³**	

← Calculate the missing volumes

c)

	Silver	Copper	Sterling silver
Density	10 g/cm³	9 g/cm³	
Mass	900g	90 g	**990 g**
Volume	90cm³	10 cm³	**100 cm³**

← Calculate the new mass and new volume by adding

d)

	Silver	Copper	Sterling silver
Density	10 g/cm³	9 g/cm³	**9.9 g/cm³**
Mass	900g	90 g	990 g
Volume	90cm³	10 cm³	100 cm³

← Calculate the new density by dividing the new mass by the new volume

Online clip

U910

Simultaneous linear equations

Component Knowledge

- Solving simultaneous linear equations with a balanced variable by elimination
- Solving simultaneous linear equations where balancing a variable is required
- Form and solve simultaneous equations.

Key Vocabulary

Simultaneous equations	Two or more equations that are two be solved (if possible) by using the *same* value for each variable
Coefficient	The number factor in an algebraic term, multiplied with variables (e.g. 4 in $4x$)
Balancing variables	Equating the coefficients of like terms in different equations by multiplying with suitable factors
Eliminating variables	Reducing the term containing a particular variable in an equation to 0 by subtracting/adding another equation with the same/opposite term
Substitution	Assigning a value to a variable (e.g. substituting $y = 8$ in $6y$ gives 48)

Solving simultaneous equations – no balancing needed

In the first example, because the two equations have **equal** terms in x – both are $3x$ – *subtracting* the equations (remember to subtract both sides) *eliminates* the x term. The resulting equation has only one unknown, y, and can be solved.

$$3x + 4y = 29$$
$$-\quad 3x + 2y = 19$$
$$2y = 10$$
$$y = 5$$

Substitute y into either equation to find x.

$$3x + (2 \times 5) = 19$$
$$3x + 10 = 19$$
$$3x = 9$$
$$x = 3$$

Here the value found for y is **substituted** into the second equation to obtain an equation in terms of x. The first equation could have been used too.

Whichever equation is used for substitution, it is good practice to check the pair of values found in the other equation too, to ensure no mistakes have been made:

$$3 \times 3 + 2 \times 5 = 19$$

In the second example, because the two equations have **opposite** terms in y – one is $2y$ and the other $-2y$ – *adding* the equations eliminates the y term.

$$3x + 2y = 16$$
$$+\quad 2x - 2y = 4$$
$$5x = 20$$
$$x = 4$$

Substitute x back in to find y.

$$(2 \times 4) - 2y = 4$$
$$8 - 2y = 4$$
$$8 = 4 + 2y$$
$$4 = 2y$$
$$2 = y$$

Forming simultaneous equations – balancing a variable

$$2x + 8y = 32$$
$$x + 3y = 13$$
×2 ×2
$$2x + 6y = 26$$
$$2y = 6$$
$$y = 3$$

$$2x + 6(3) = 26$$
$$2x + 18 = 26$$
$$2x = 8$$
$$x = 4$$

Here neither the x nor the y terms are already balanced. But the x terms can be balanced by multiplying the second equation by 2. (Remember to **multiply both sides** by the factor.)
The modified second equation can then be subtracted from the first, and the subsequent steps are as before.

$$5x + 4y = 19$$
$$2x - 3y = 3$$
×3 ×4 ×4 ×3
$$8x - 12y = 12$$
$$15x + 12y = 57$$
$$23x = 69$$

$$x = 3$$

$$15 + 4y = 19$$
$$4y = 4$$
$$y = 1$$

In this example the y terms can be balanced by multiplying the first equation by 3 and the second by 4, since 12 is the lowest common multiple of the starting coefficients. (Alternatively, we can balance the x terms. What factors would be needed in that case?)
The modified equations are then added – since the y terms have opposite signs – and the following steps are as before.

Forming simultaneous equations to solve a problem

Barry buys 200 pieces of stationery for £76.

Of the 200 pieces of stationery, x of them are rulers that cost 50p each and y of them are pens that cost 20p each.

Find how many rulers and pens Barry buys.

The information in the question can be written as the simultaneous equations

$x + y = 200$

$50x + 20y = 7600$ (amounts are written in pence)

Multiply the first equation by 50 to give $50x + 50y = 10000$. The x terms are now balanced, and subtracting the second equation gives $30y = 2400$.

Therefore $y = 80$, and using the first equation $x = 120$.

Online clips

U760

Factorising Quadratics

Component Knowledge

- Be able to factorise a quadratic of the form $ax^2 + bx + c$ when $a = 1$
- Be able to factorise a quadratic of the form $ax^2 + bx + c$ when $a \neq 1$
- Factorise a quadratic using the difference of 2 squares
- To use factorising to solve a quadratic equation

Key Vocabulary

Quadratic expression	Equation of the form $ax^2 + bx + c$, where a, b and c are any form of number
Coefficient	Number of front of a letter, e.g. the coefficient of x^2 in the term $-5x^2$ is -5
Factor	A common number or letter that will divide into a term
Factorise	An expression written as a product of it's factors
Product	Multiplication of two or more values

Factorising when a = 1

Factorise

$$x^2 + 6x + 8$$

We need two numbers with a product of +8 and a sum of +6,

so we list all of the products and check their sum

$1 \times 8 = 8 \qquad 1 + 8 = 9$

$2 \times 4 = 8 \qquad 2 + 4 = 6$

This is the correct product/sum pair

We then re-write the quadratic as

$$x^2 + 2x + 4x + 8$$

Factorising each half separately gives

$$x(x + 2) + 4(x + 2)$$

Taking out the common factor of the bracket then gives

$$(x + 2)(x + 4)$$

Difference of two squares

Look out for this specific case where

$$a^2 - b^2 = (a + b)(a - b)$$

Remember that this won't work if it contains + instead of −.

1) Factorise $x^2 - 25 = (x + 5)(x - 5)$ or $(x - 5)(x + 5)$

(Expanding gives $x^2 - 5x + 5x - 25 = x^2 - 25$.)

Sometimes there could be more than one variable (letter) in the expression.

2) $9x^2 - y^2 = (3x + y)(3x - y)$ or $(3x - y)(3x + y)$

3) $25c^2 - 16d^2 = (5c + 4d)(5c - 4d)$ or

$(5c - 4d)(5c + 4d)$

Factorising when a ≠ 1

1) Factorise $2x^2 + 11x + 12$.

Here you need two numbers with a product of +24 (from +2 × +12) and a sum of +11.

The two numbers are +3 and +8.

Re-write the expression using your two numbers (in either order) to replace the middle term.

(Note that this time you don't put them straight into brackets!)

$2x^2 + 3x + 8x + 12$ or $2x^2 + 8x + 3x + 12$

Factorise a pair of terms at a time, by taking out common factors. (Make sure the 'introduced brackets' contain identical terms.)

$x(2x + 3) + 4(2x + 3)$ or $2x(x + 4) + 3(x + 4)$

Write down the 'repeated' bracket, then construct a second bracket using 'everything else'.

You then have:

$(2x + 3)(x + 4)$ or $(x + 4)(2x + 3)$

Factorising when a ≠ 1 (involving negatives)

Factorise

$2x^2 - 5x - 12$

We need two numbers with a product of +24 and a sum of +11,

so we list all of the products and check their sums

$2 \times 12 = 8 \quad 2 + 12 = 9,\ -2 + 12 = 10,\ 2 + (-12) = -10$

$4 \times 6 = 24 \quad 4 + 6 = 10,\ -4 + 6 = 2,\ 4 + (-6) = -2$

$1 \times 24 = 24 \quad 1 + 24 = 25,\ -1 + 24 = 23,\ 1 + (-24) = -23$

$3 \times 8 = 24 \quad 3 + 8 = 11, -3 + 8 = 5,\ 3 + (-8) = -5$

This is the correct product/sum pair

We then re-write the quadratic as

$2x^2 - 8x + 3x - 12$

Factorising each half separately gives

$2x(x - 4) + 3(x - 4)$

Taking out the common factor of the bracket then gives

$(x - 4)(2x + 3)$

Solving Quadratics by factorisation

You must be able to factorise quadratics in order to solve quadratic equations using this method.

Example 1

Solve $x^2 + 6x + 5 = 0$

This factorises into $(x + 5)(x + 1) = 0$

Each bracket needs to equal 0

$x + 5 = 0$ or $x + 1 = 0$

x = -5 or **x = -1**

Example 2

Solve $x^2 + 3x - 10 = 0$

This factorises into $(x + 5)(x - 2) = 0$

$x + 5 = 0$ or $x - 2 = 0$

x = -5 or **x = 2**

Example 3

Solve $x^2 - 6x + 9 = 0$

This factorises into $(x - 3)(x - 3) = 0$

This equation has repeated roots

$(x - 3)^2 = 0$

This means there is only one solution, **x = 3**

Further useful information

Check first that you can expand double brackets (using any appropriate method, such as using a grid or 'FOIL') e.g.

1. $(x + 1)(x - 6) = x^2 - 6x + x - 6$
   $= x^2 - 5x - 6$
2. $(2x - 3)(x + 4) = 2x^2 + 8x - 3x - 12$
   $= 2x^2 + 5x - 12$

Avoid being caught out!

- Sometimes a quadratic expression doesn't require double brackets e.g. $2x^2 - 7x = x(2x - 7)$
- Sometimes you can start by taking out a common factor e.g. $2x^2 - 72 = 2(x^2 - 36) = 2(x + 6)(x - 6)$

Online clips

U178, U858, U228, U960, U963

Column vectors

Component knowledge

- Understand that vectors are a way of showing the magnitude (size) and direction an object moves (translates).
- Represent vectors
- Add, subtract and multiply vectors

Key Vocabulary

Vector	A vector has magnitude (size) and direction
Magnitude	Size of an object- can be a distance or quantity
Scalar	A scalar on has a magnitude (size) and no direction
Constant	A variable that remains the same

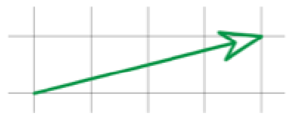
Vectors

Vectors are often written as column vectors

Left or right → $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ ← Up or down

Positive values are right and up. Negative values are left and down.
This is 3 right and 4 down.

This is the vector $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$



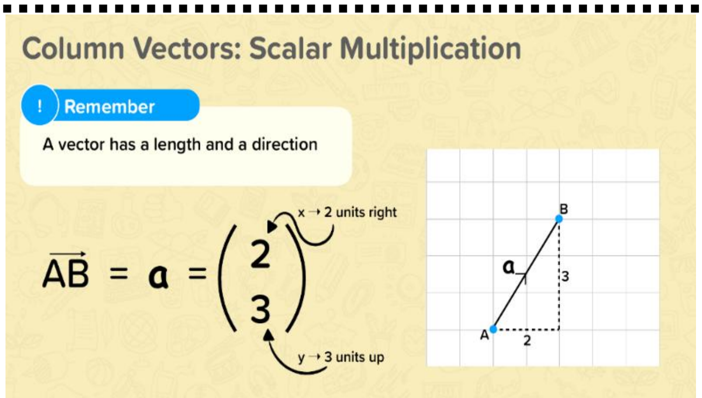
It goes 4 units right and 1 unit up.

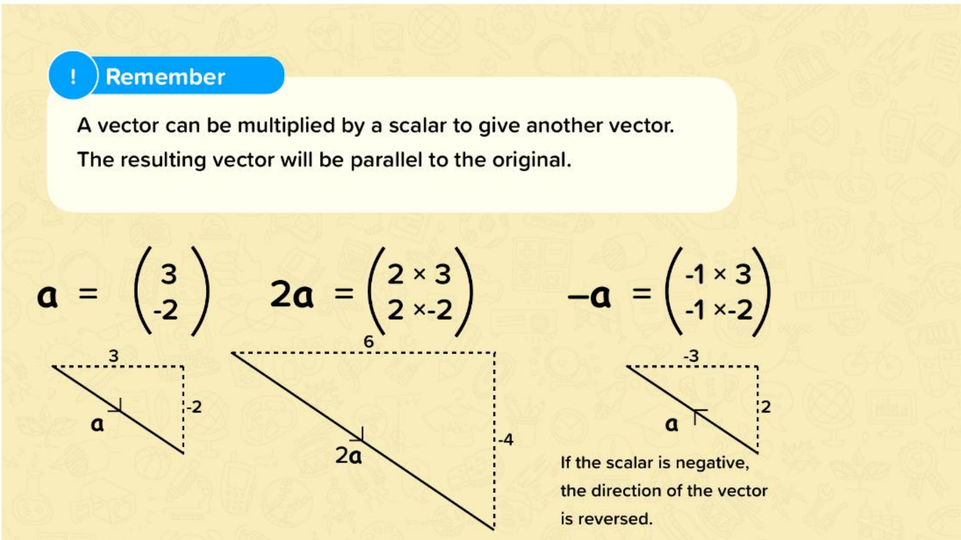
Add/subtract vectors:

$$\begin{pmatrix} 8 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

Multiply vectors by a constant

$$3\begin{pmatrix} 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 12 \\ 21 \end{pmatrix}$$





Online clips

U632, U903, U564

Transformations

Component Knowledge

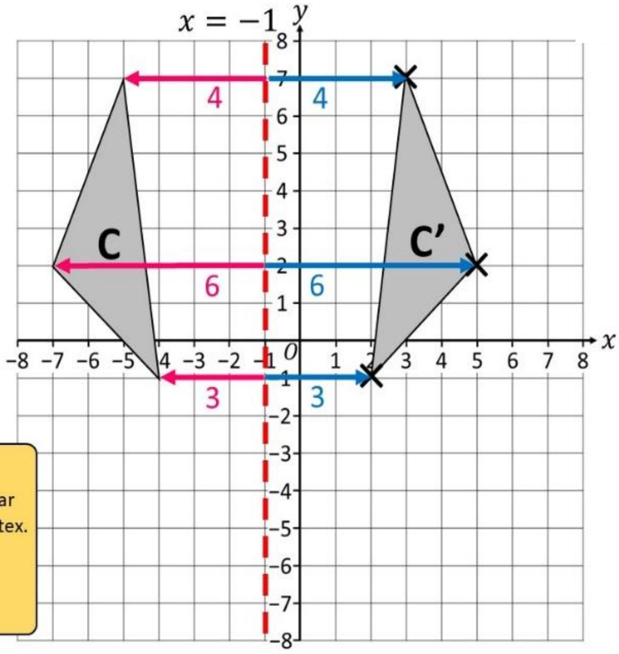
- Rotate, reflect and translate a shape.
- Describe a rotation, reflection and translation.

Key Vocabulary

Rotation	The turning of a shape around a fixed point.
Reflection	An image of a shape as it would be seen in a mirror.
Perpendicular	At a right angle to a point or line.
Translation	Moving every point by the same distance in a given direction.
Vertex	Corner of a shape- where two lines meet in a polygon.

Reflection:

Reflect shape **C** in the line $x = -1$. Label the new shape **C'**.



1) Plot the line.
2) Count squares perpendicular from the line to each vertex.
3) Plot each vertex an equal distance away on the opposite side.

Information needed to perform a reflection:

- Mirror line. This usually an equation e.g. y=2, x= -2.

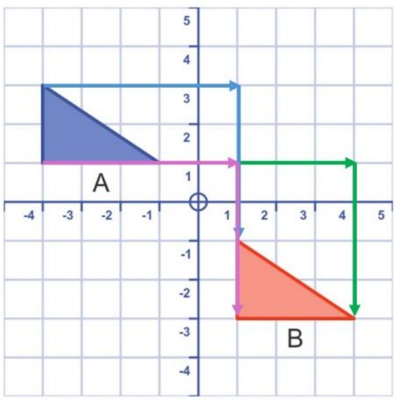
Translation:

Translate shape A by the vector $\binom{5}{-4}$

Move each vertex 5 right.

$$\binom{5}{-4}$$

Move each vertex 4 down.



$\binom{x}{y}$

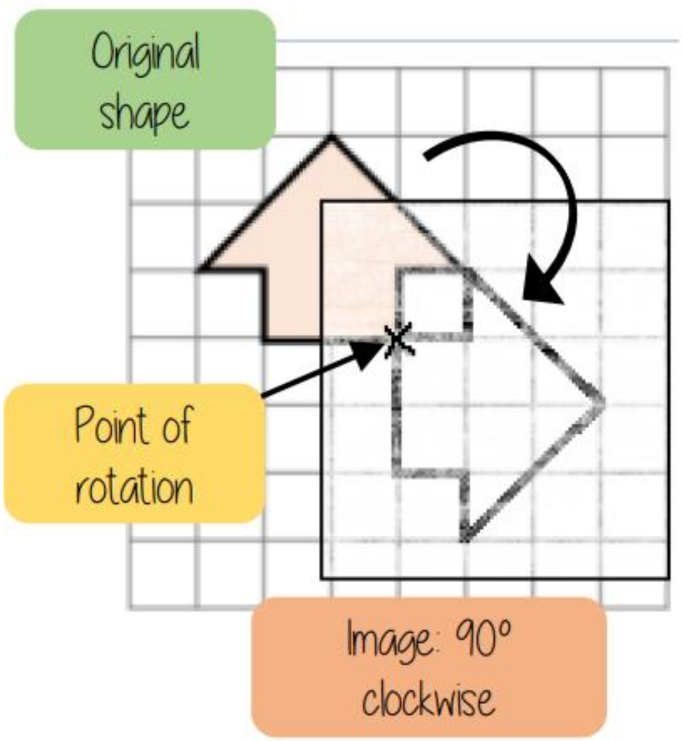
x → Positive-Right, Negative - Left

y → Positive-Up, Negative - Down

Information needed to perform a reflection:

- Vector. This is usually as a column vector e.g. $\binom{3}{-7}$

Rotation:



1. Trace the original shape (mark the point of rotation)

2. Keep the point in the same place and turn the tracing paper

3. Draw the new shape

Information needed to perform a rotation:

- Centre of rotation. This is usually a co-ordinate.
- Direction of rotation. Either Clockwise or anti-clockwise.
- Degrees of rotation. 90° or 180° or 270°
- Tracing paper.

Online clips

M910, M290, M139

Enlargement

Component Knowledge

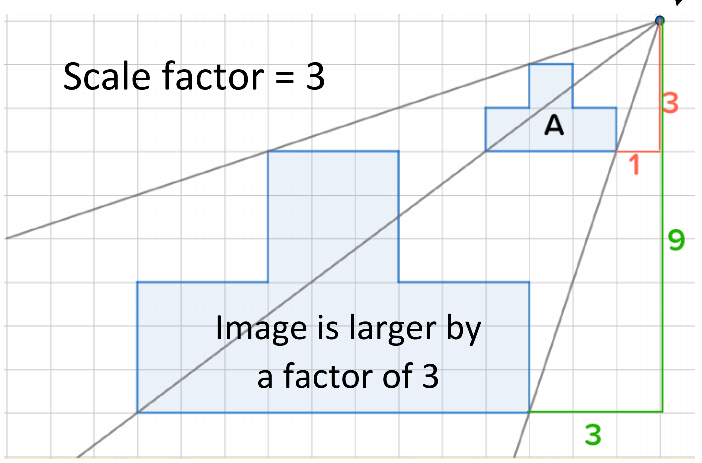
- Enlarge a rectilinear shape by a given positive scale factor
- Enlarge a rectilinear shape, given a positive integer scale factor and a centre
- Enlarge a rectilinear shape, given a positive fractional scale factor and a centre
- Enlarge a rectilinear shape, given a negative scale factor and a centre

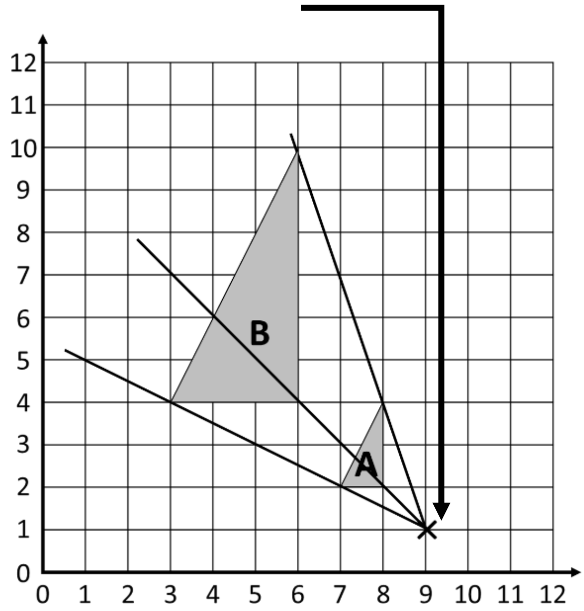
Key Vocabulary

Enlargement	A transformation of a shape in which all dimensions are multiplied by the same number
Scale factor	The number by which dimensions are multiplied in an enlargement
Centre of enlargement	The point from which distances to the *object(original shape)* and the *image (new shape)* of an enlargement are measured

Enlarging by a positive *integer* scale factor from a centre

Centre of enlargement





Measure the distance from the centre of enlargement to each vertex of the *object* shape A; the corresponding vertex in the *image* is triple that distance in the **same** direction

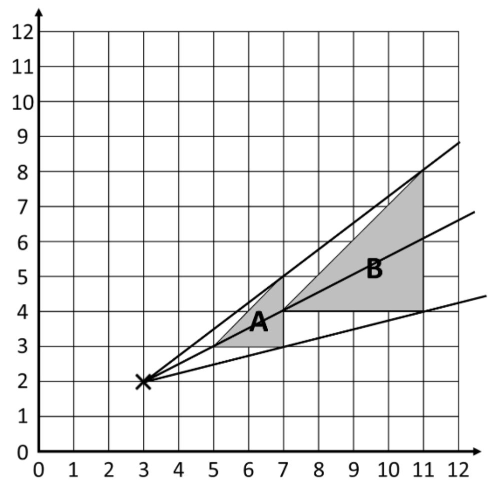
If the object shape is drawn on a coordinate grid, the centre may be specified by coordinates (here the centre is (9,1))

Describing an enlargement

To determine the scale factor, calculate the ratio of the lengths of corresponding sides in the object and its image.

For the centre, draw lines through two pairs of corresponding vertices and find their point of intersection

The enlargement shown here – from A to B – has scale factor 2 and centre (3,2)

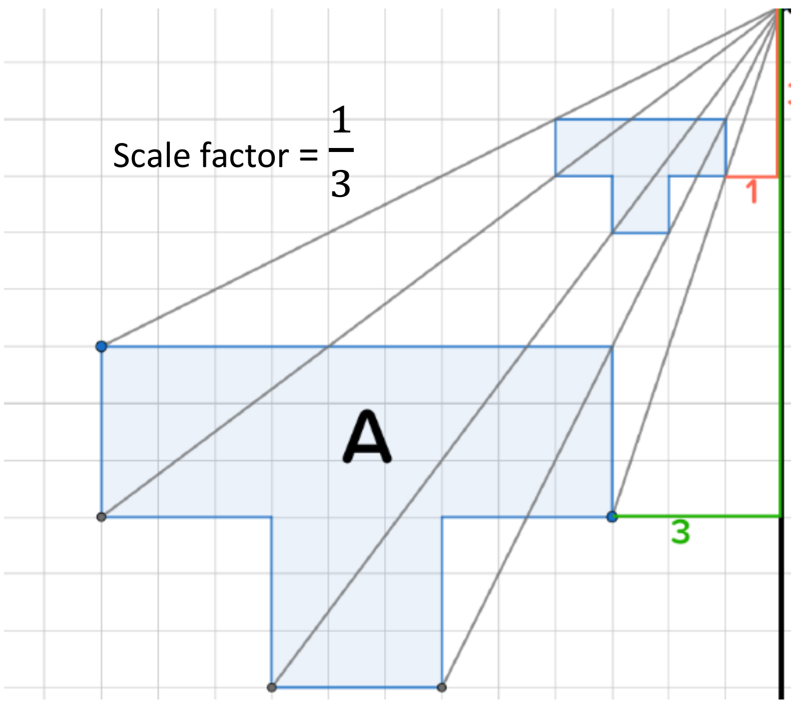


Enlarging by a positive *fractional* scale factor from a centre

A positive scale factor that is smaller than 1 reduces the dimensions of the object shape.

Here the distance from the centre of enlargement to each vertex of the object shape A is measured and then multiplied by $\frac{1}{3}$ (**divided** by 3) to find the corresponding vertex in the image (still in the same direction)

Image lengths are a third of the length of the object's (shape has got smaller)

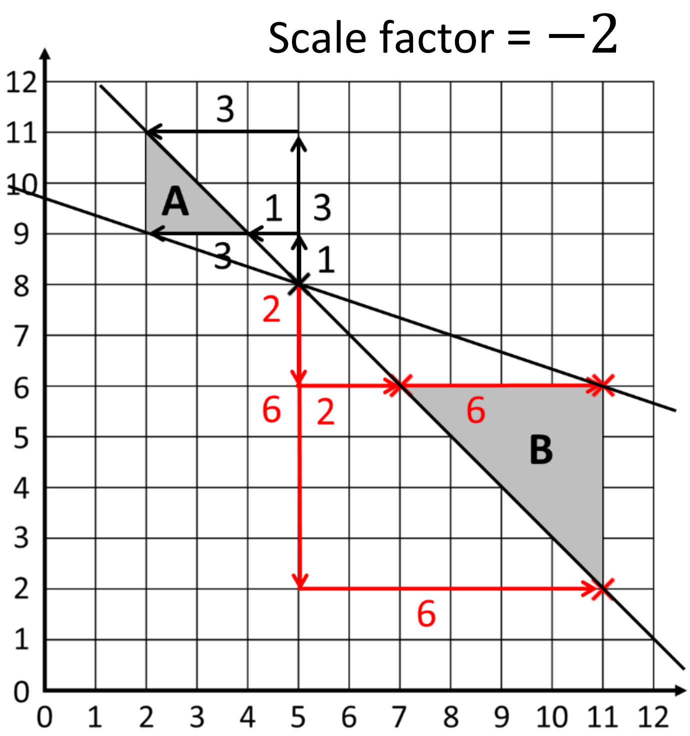


Enlarging by a *negative* scale factor from a centre

In enlargement by a *negative* scale factor, the object and its image are at opposite directions *from the centre.*

Here the distance from the **centre of enlargement** $(5, 8)$ to each vertex of the object shape A is measured and then multiplied by 2 to find the distance to the corresponding vertex in the image B, but in the **opposite direction.**

Note that in this case the image is inverted as well as enlarged.



Online clips

U519, U134

Scatter Graphs

Component Knowledge

- Plot points on a scatter graph
- Describe the relationship between variables using a scatter graph
- Identify outliers on a scatter graph
- Draw and interpret a line of best fit

Key Vocabulary

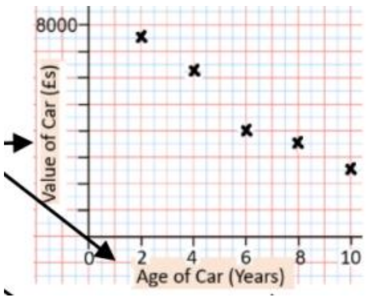
Origin	Where two axes meet on a graph
Outlier	A point that lies outside the trend of the graph
Relationship	The link between two variables
Correlation	The mathematical definition for the type of relationship between two variables
Line of best fit	A straight line on a graph that represents the data on a scatter graph
Interpret	Describe what the data is showing

Plotting a scatter graph

Age of car (years)	2	4	6	8	10
Value of car (£)	7500	6250	4000	3500	2500

The data forms information pairs for the scatter graph that you plot as coordinates e.g. (2, 7500).

Make sure axes are clearly labeled and values are equally spaced

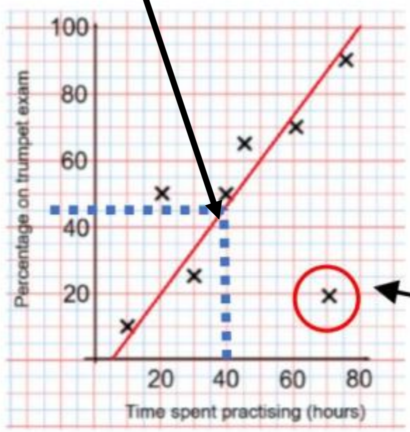


The line of best fit

We use the line of best fit to estimate other values.

E.g. 40 hours revising predicts 45% score on exam

We cannot use our line of best fit to predict information outside of our data range.



This point is an 'outlier'
It doesn't fit the model and stands apart from the rest of the data

Types of Correlation- describes the relationship only.

Positive correlation
As one variable increases so does the other variable.

Negative correlation
As one variable increases the other variable decreases.

No correlation
There is no relationship between the two variables.

Online clips

M769, M596

Venn Diagrams

Component Knowledge

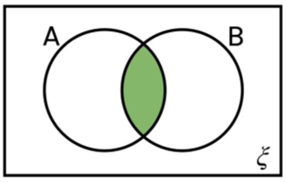
- Complete a Venn Diagram when given a set of data
- Fill in missing values in a Venn Diagram
- Interpret a Venn diagram
- Find probabilities from a Venn Diagram
- Use simple set notation

Key Vocabulary

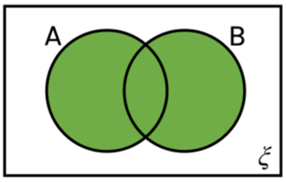
Set	A collection of “things” (objects or numbers)
Union	The set made by combining the elements of two sets
Intersection	The intersection of two sets has only elements common to both sets
Probability	The change that something happens
Venn Diagram	A diagram that shows sets which elements belong to which set by drawing regions around them. It is used to represent data that has an overlap.

Key Concepts

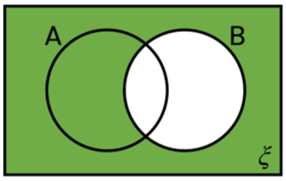
Venn diagrams show all possible relationships between different sets of data.



$A \cap B$
The **intersect** of A and B.
The set of elements in **both A and B**.



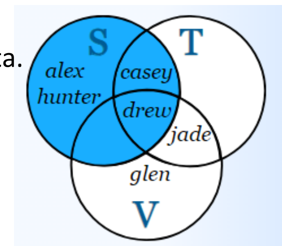
$A \cup B$
The **union** of A and B.
The set of elements in **A or B or both.**



B’
The **complement** of B.
The set of elements **not in B.**

Venn Diagrams with 3 sets

Diagrams can be drawn to show more than 2 sets of data. This is an example of a Venn Diagram containing 3 sets.



S = {Alex, Hunter, Casey and Drew}

T = {Jade, Casey and Drew}

V = {Drew, Jade and Glen}

Example

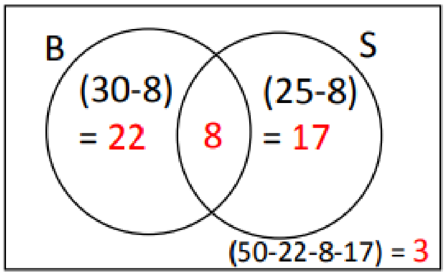
Out of 50 people surveyed:

30 have a brother

25 have a sister

8 have both a brother and a sister

This is what the Venn Diagram for this information would look like



Remember – the people in the intersection are also included in the whole circle so we don’t duplicate data.

From the Venn Diagram, we can see that the probability of someone from this group just having a brother is 22/50.

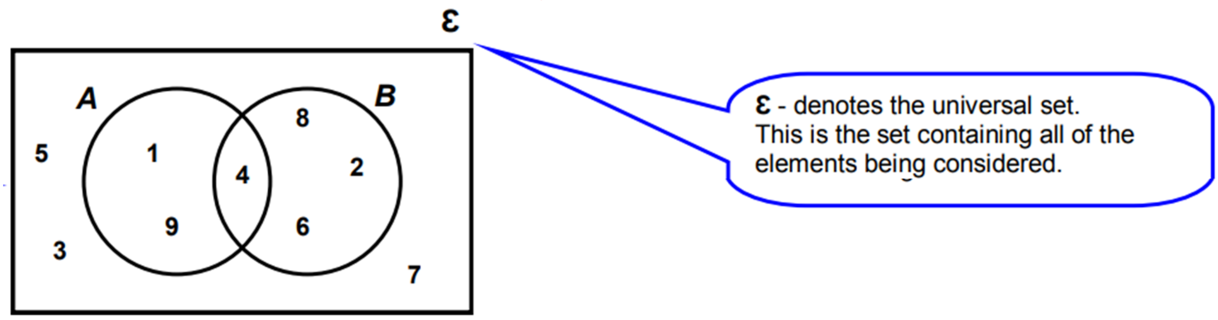
The probability of someone from this group having neither a brother or a sister is 3/50.

The probability of having a brother and a sister,

$$P(A \cap B) = \frac{8}{50}$$

Example: Given a set of numbers

Ɛ={1,2,3,4,5,6,7,8,9}
A={square numbers}
B={even numbers}



In set A 'the square numbers' are 1, 4 and 9.

In set B the 'even numbers' are 2, 4, 6, 8.

4 is in both groups so would go in the centre (the intersection)

Outside of the circles are any numbers remaining in Ɛ

Online clips

M829, M419, M834