Direct and Inverse proportion

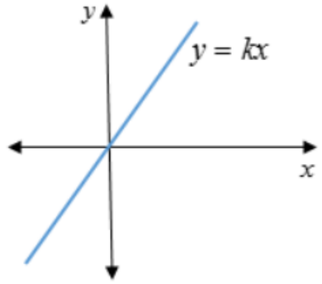
Component Knowledge

- Find the constant of proportion that connects two variables.
- Find an equation connecting two variables.
- Find missing value using the equation of proportion.
- Combine two equations of proportion

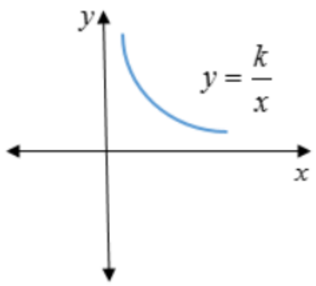
Key Vocabulary

<table>
<tr>
<td>Direct Proportion</td>
<td>If two quantities are in direct proportion, as one increases, the other increases by the same rate.

If y is directly proportional to x, this can be written as $\boldsymbol{y \propto x}$

An equation of the form $\boldsymbol{y = kx}$ represents direct proportion, where k is the constant of proportionality.</td>
<td>
</td>
</tr>
<tr>
<td>Inverse Proportion</td>
<td>If two quantities are inversely proportional, as one increases, the other decreases by the same percentage.

If y is inversely proportional to x, this can be written as $\boldsymbol{y \propto \frac{1}{x}}$

An equation of the form $\boldsymbol{y = \frac{k}{x}}$ represents inverse proportion.</td>
<td>
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Direct: **y = kx** or **y** ∝ **x**

Inverse: **y** = $\frac{k}{x}$ or **y** ∝ $\frac{1}{x}$

1. **Solve to find k** using the pair of values in the question.
2. **Rewrite the equation** using the k you have just found.
3. **Substitute the other given value** from the question in to the equation to **find the missing value**.

Direct Proportion

p is directionally proportional to q.
When p = 12, q = 4.
Find p when q = 20.

Step 1 - Write down the formula using k as the constant.

$$p = kq$$

Step 2 - Substitute in the values of y and x.

$$12 = 4k$$

Step 3 - Solve the equation to find the value of k .

$$k = 3$$

Step 4 - Re-write the original equation substituting k for the actual value .

$$p = 3q$$

Step 5 - Substitute the new radius into this equation.

$$p = 3 \text{ x } 20 = 60$$

Inverse Proportion

The height h cm of a plastic cylinder is inversely proportional to its radius r cm.
A plastic cylinder of height 6cm has a radius of 4 cm.
Work out the height of a cylinder with a radius of 3cm.

Step 1 - Write down the formula using k as the constant.

$$h = \frac{k}{r}$$

Step 2 - Substitute in the values of y and x.

$$6 = \frac{k}{4}$$

Step 3 - Solve the equation to find the value of k .

$$k = 24$$

Step 4 - Re-write the original equation substituting k for the actual value .

$$h = \frac{24}{x}$$

Step 5 - Substitute the new radius into this equation.

$$h = \frac{24}{3} = 8cm$$

Online clips

U721, U357, U640, U407, U364, U138

Finding and using the nth term

Component Knowledge

- Find the common difference between terms in a sequence
- Using the common difference to find the nth term
- Using the nth term to find terms in a sequence

Key Vocabulary

Sequence	A list of numbers or objects in a special order
Linear	A sequence where each term is added, or subtracted, by the same amount each time.
Pattern	Objects or numbers that are arranged following a rule or rules
Nth Term	A formula that enables us to find any term in a sequence
Term	In algebra, a term is either a single number or variable, or numbers and variables multiplied together.

How to find common differences & the nth term of a linear sequence

The nth term is the general rule for a sequence. We can use the nth term to then calculate any term in the sequence.

Here is a sequence: 5, 8, 11, 14, ...

1. Find the difference between the numbers.

5, 8, 11, 14

+3 +3 +3 = 3n

A difference of +3 means we need to look at the +3 times table.

2. Calculate how you get from the times table to the original sequence.

3, 6, 9, 12, ...

+ 2

5, 8, 11, 14, ...

The nth term is 3n + 2.

We can also write this as

$t_n = 3n + 2$

Decreasing sequences – follow the same steps but your nth term will be negative

A difference of -3 means we need to look at the -3 times table.

-3 -3 -3 -3

-5, -8, -11, -14, -17

-2 -2

-3n: -3 -6 -9 -12 -15

Calculate how you get from the times table to the original sequence

-3n -2

Using the nth term to create a sequence

Write the first five terms of the sequence $3n + 4$.

n represents the position in the sequence. The first term in the sequence is when $n = 1$, the second term in the sequence is when $n = 2$, and so on.

To find the terms, **substitute** n for the position number:

- when $n = 1$, $3n + 4 = 3 \times 1 + 4 = 3 + 4 = 7$
- when $n = 2$, $3n + 4 = 3 \times 2 + 4 = 6 + 4 = 10$
- when $n = 3$, $3n + 4 = 3 \times 3 + 4 = 9 + 4 = 13$
- when $n = 4$, $3n + 4 = 3 \times 4 + 4 = 12 + 4 = 16$
- when $n = 5$, $3n + 4 = 3 \times 5 + 4 = 15 + 4 = 19$

The first five terms of the sequence: $3n + 4$ are $7, 10, 13, 16, 19, \ldots$

Using the nth term to find if a number is in a sequence

Is the number 14 in the sequence 4n + 2?

$$4n + 2 = 14$$
$$-2 \qquad -2$$
$$4n = 12$$
$$\div 4 \qquad \div 4$$
$$n = 3$$

If you get a decimal here, then the term isn't in the sequence

Yes, 14 is the 3rd term in the sequence.

Online clips

M381, M241, M166, M991

Geometric Sequences

Component Knowledge

- Identify when a sequence is geometric
- Find next terms in a geometric sequence
- Find the nth term of a geometric sequence
- Use the nth term of a geometric sequence to find missing terms

Key Vocabulary

Term	Each value in a sequence is called a term
Sequence	A number or picture pattern with a specific rule
Geometric sequence	Has a common multiple between each term
Nth term	A rule that allows you to calculate the term in the nth position of the sequence

A geometric sequence if where you have to multiply one term by a constant ratio (r) to get to the next

Ratio of 2 – If the ratio is positive the sequence will increase

x2 x2 x2 x2 x2

5 , 10 , 20 , 40 , 80 , 160 , … Here the first term is 5 and the ratio r is 2

Ratio of 0.5 – if the ratio is a decimal the sequence will decrease

x0.5 x0.5 x0.5 x0.5 x0.5

8 , 4 , 2 , 1 , 0.5 , 0.25 , … Here the first term is 5 and the ratio r is 2

Ratio of -3 – if the ratio is a decimal the sequence will flip between negative and positive numbers

x-3 x-3 x-3 x-3

5 , -15 , 45 , -135 , 405 , … Here the first term is 5 and the ratio r is 2

Finding the nth term

$$nth\ term = ar^{n-1}$$

Where

a= the first term

r = the common ratio

Example:

Find the nth term of

4 , 12 , 36 , 108 , …

x 3 x 3 x 3

Here each term is being multiplied by 3 so the common ratio r=3

The first term a=4

So the nth term of the sequence is:

$$4 \times 3^{n-1}$$

Online clips

Using the nth term of a Geometric sequence

Example

Find the 10th term of the sequence

3 , 6 , 12 , 24 , …

Here each term is being multiplied by 2

So r=2

And the first term a=3

So the nth term is

$$3 \times 2^{n-1}$$

To find the 10th term we substitute in n=10

$$3 \times 2^{10-1}$$

$$3 \times 2^{9} = 3072$$

Quadratic Sequence

Component Knowledge

- To be able to find the nth term of a quadratic sequence

Key Vocabulary

Quadratic	Involving the second power (i.e. involving squared numbers/values)
Coefficient	A number quantity placed before and multiplying the variable in an expression.
Nth term	The position of a term of the sequence, e.g. n = 1 means the first term etc

The simplest quadratic sequence is the list of square numbers and has the nth term of n^2, 1, 4, 9, 16, 25, ….. .

For a more complicated quadratic sequence have a different amount between each term but the difference between these, known as the second difference, is constant.

e.g. Sequence 1 3 6 10 15 21

1st difference +2 +3 + 4 +5 + 6

2nd difference +1 +1 +1 +1

The second difference is always double the amount of n^2 in the nth term. i.e. if you need to find the nth term halve the second difference and use that as the coefficient of n^2.

To find the full nth term of a quadratic sequence,

- Find the coefficient of n^2 (half the second difference)
- Multiply the value of n^2 for each term by this coefficient and subtract from the original sequence.
- Find the nth term of the remaining sequence.

For example, Find the nth term of the sequence 5, 7, 11, 17, 25, ….

Sequence 5 7 11 17 25

1st difference +2 +4 +6 +8

Second difference +2 +2 +2

The second difference is 2, half of this gives one lot of n^2.

n	1	2	3	4	5
Sequence	5	7	11	17	25
n^2	1	4	9	16	25
Sequence minus n^2	4	3	2	1	0

The nth term of 4, 3, 2, 1, 0 is $-n + 5$ (see nth term of linear sequences)

Therefore, the nth term of the quadratic sequence is $\mathbf{n^2 - n + 5}$

Online clips

Histograms

Component Knowledge

- To calculate frequency density
- To construct a histogram

Key Vocabulary

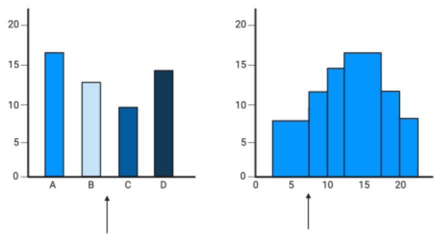
Continuous data	Continuous data can take any value (within a range) and can be measured. Examples include height, weight, length and time.
Frequency	The number of times an event occurs
Frequency density	The number of times an event occurs, **within a given interval**
Class Interval	A group values that data belongs to, e.g. 10-19, 20-29, $10 < x \leq 20$ or $20 < x \leq 30$
Class width	The difference between the bounds of the class or group, (the range of values that grouped data belongs to)

Basics:

A **histogram** looks similar to a **bar chart**, with some key differences.

- Bars **do not have to be**, but can be, **equal in width**
- Bars must have **no gaps** between them
- The x axis is a **continuous scale**.
- The y axis is **frequency density** rather than frequency

Bar Chart v Histogram



Gaps between bars of equal width.

No gaps between bars of equal or unequal width.

Calculating Frequency Density:

Construct a histogram of the following data

Time (mins), t	Frequency
$0 \leq t < 10$	60
$10 \leq t < 15$	40
$15 \leq t < 20$	75
$20 \leq t < 50$	150

Firstly we need to calculate frequency density, in this case the frequency per minute

Generally (not always):

$$Frequency\ density = \frac{frequency}{class\ width}$$

Time (t) mins	Frequency	Class width	Frequency density
$0 \leq t < 10$	60	10	$60 \div 10 = 6$
$10 \leq t < 15$	40	5	$40 \div 5 = 8$
$15 \leq t < 20$	75	5	$75 \div 5 = 15$
$20 \leq t < 50$	150	30	$150 \div 30 = 5$

Class width is the difference between the 2 bounds.

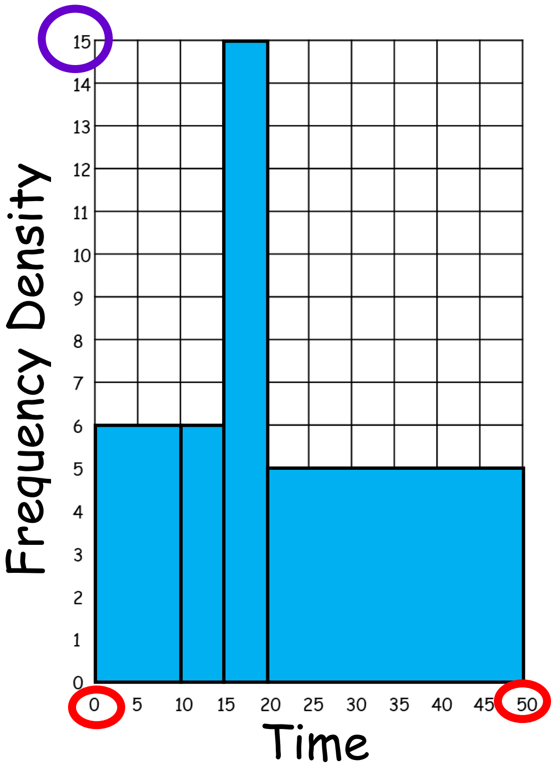
For $20 \leq t < 50$, *the class width is* $50 - 20 = 30$

Frequency density is plotted on the y axis our histogram (vertical axis), rather than just **frequency**.

Constructing Histograms:

Given our table and calculated frequency densities

Time (t) mins	Frequency	Class width	Frequency density
0 ≤ t < 10	60	10	60 ÷ 10 = 6
10 ≤ t < 15	40	5	40 ÷ 5 = 8
15 ≤ t < 20	75	5	75 ÷ 5 = 15
20 ≤ t < 50	150	30	150÷ 30 = 5



Our x axis, for **time**,

needs to go from 0 to 50

Our largest **frequency density**,

for the y axis, is 15

Then draw bars of correct width to the necessary height.

Key feature:

Unlike a bar chart where the height of each bar represents the frequency.

In a histogram the **area of each bar represents the frequency**.

WARNING:

The frequency density is not always $= \frac{frequency}{class\ width}$ especially when class intervals are large.

eg

Cells	Frequency	~~Frequency density~~	Frequency density
0 ≤ c < 1000	5	~~5 ÷ 1000 = 0.005~~	5 ÷ *1* = 5
1000 ≤ c <5000	8	~~8 ÷ 4000 = 0.002~~	8 ÷ *4* = 2
5000 ≤ c < 20 000	30	~~30 ÷ 15000 = 0.002~~	30 ÷ *15* = 2
20 000 ≤ c < 50 000	120	~~120 ÷ 30000 = 0.004~~	120 ÷ *30* = 4

Here we found the frequency per ***thousand cells***, rather than per 1 cell

Online clips

U185, U814

Histograms - interpreting

Component Knowledge

- To calculate frequency from frequency density
- To understand area of bar represents frequency

Key Vocabulary

Frequency	The number of times an event occurs
Frequency density	The number of times an event occurs, **over a given interval**
Class Interval	A group values that data belongs to, eg 10-19, 20-29, $10 < x \leq 20$ or $20 < x \leq 30$
Class width	The difference between the bounds of the class or group, (the range of values that grouped data belongs to)

Key feature:

Unlike a bar chart where the height of each bar represents the frequency.

In a histogram the **area of each bar represents the frequency**.

Calculating Frequency Density from Histogram:

Complete the frequency table, and the histogram, for the data shown

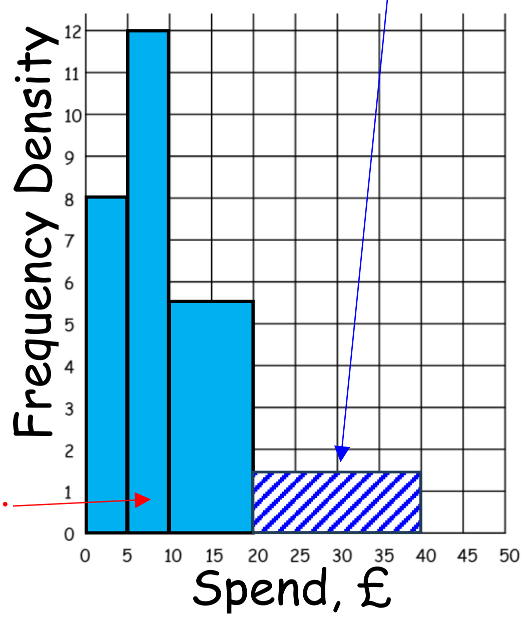
Spend (£), s	Frequency
$0 \leq s < 5$	40
$5 \leq s < 10$	60
$10 \leq s < 20$	55
$20 \leq s < 40$	30

For $20 \leq s < 40$, our bar needs to be 20 units wide and have an area of 30.

Area (frequency) = 1.5x20 = **30** so we need to draw a bar 1.5 units tall between 20 and 40

First we need to identify a **known pair** (frequency in the table & bar in the histogram)

We can see for the group who spent $5 \leq s < 10$, there is a frequency of 60 and a bar that is 5 units wide and 12 units tall.



Remember: The **area** of our **bar** is representative of the **Frequency**

The area of our $5 \leq s < 10$ bar is 5 x 12 = 60, so here area **equals** frequency

$0 \leq s < 5$, has a bar that is 5 units wide and 8 units tall.

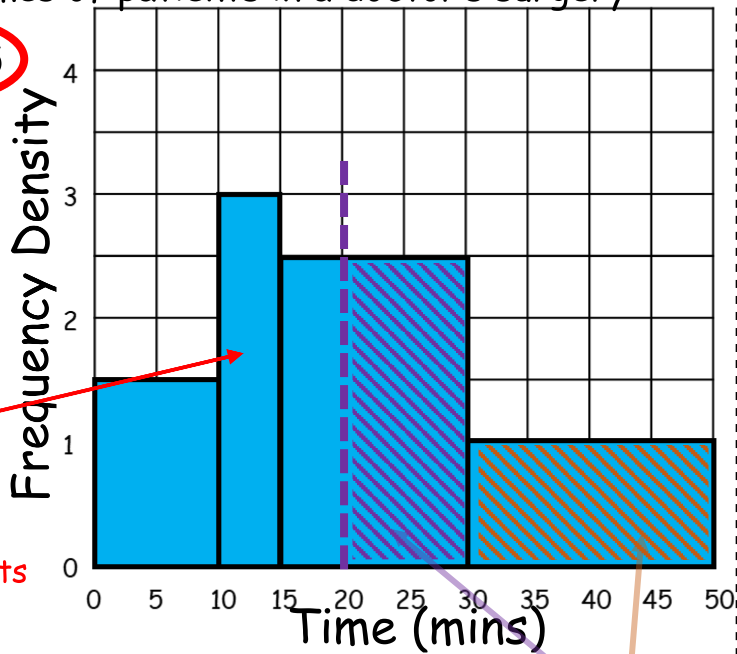
Area (frequency) = 5x8 = **40**

$10 \leq s < 20$, has a bar that is 10 units wide and 5.5 units tall.

Area (frequency) = 5.5x10 = **55**

Interpreting Histograms:

The histogram shows the waiting times of patients in a doctor's surgery



30 people waited between 10 and 15 minutes.

Estimate many people waited for more than 20 minutes?

First we need to identify a **known pair** (frequency in the table & bar in the histogram)

The area of this bar, that is 5 units wide and 3 units tall is 5 x 3 =15, this represents a frequency of 30.

So we need to find the area of the bars after 20mins and double this to get the frequency. Section the diagram at 20 minutes.

From 20 to 30 the bar in 10 units wide and 2.5 units tall. Area = 10x2.5 = 25

From 30 to 50 the bar is 20 units wide and 1 unit tall, Area = 20x1 = 20

This gives us a total area of 20 + 25 = 45 which we need to double to get our frequency, 45 x 2 = **90 patients** waited for more than 20 minutes.

Alternative ways to interpret:

- You could be asked to find the number between two values on the x axis
- You could be asked to estimate the median where you would need to find the point on the x axis that splits your entire area of bars in half

Online clips

U983, U267

Simplifying Algebraic Fractions

Component Knowledge

- Application of the law of indices
- Factorising algebraic terms
- Factorising quadratics
- Recognise the difference of two squares

Key Vocabulary

Algebraic Fractions	A fraction whose numerator and denominator are algebraic expressions
Factorise	Using Highest Common Factors (HCF) to simplify an expression
Simplify	Reducing the expression/fraction to a simpler form.

Simplifying Algebraic fractions by finding and cancelling common factors:

Recap: Law of Indices

$\frac{a^m}{a^n} = a^m \div a^n$

Law of indices a^{m-n}

Example 1: Simplify

$$\frac{b^6}{b^4} = \frac{b \times b \times b \times b \times b \times b}{b \times b \times b \times b}$$

This can be simplified by 'cancelling out common factors'

Such that $b^6 \div b^4$

You can use the rule b^{6-4}

$= b^2$

Example 2: Simplify

$$\frac{a^2b}{ab^2}$$

This can be simplified by 'cancelling out common factors'

$$\frac{a^2b}{ab^2} = \frac{a \times a \times b}{a \times b \times b}$$

$$= \frac{a}{b}$$

Simplify

$$\frac{16e^3f^7}{8e^5f^4}$$

$$= \frac{16 \times e \times e \times e \times f \times f \times f \times f \times f \times f \times f}{8 \times e \times e \times e \times e \times e \times f \times f \times f \times f}$$

$$= \frac{2f^3}{e^2} \text{ or } = 2e^{-2}f^3$$

Example 3: Write this fraction in its simplest form:

$$\frac{x+2}{(x+5)(x+2)}$$

$x + 2$ is a common factor on the numerator and denominator

$$= \frac{x+2}{(x+5)(x+2)}$$

$$= \frac{1}{(x+5)}$$

Note: When simplifying common factors, you may need to write/place hold with 1, as dividing a number by itself gives 1

Simplifying Algebraic fractions by factorising:

Example 1:

Simplify

$$\frac{x^2+7x+10}{x+5}$$

$$=\frac{(x+2)(x+5)}{x+5}$$

$$=\frac{(x+2)\cancel{(x+5)}^{1}}{\cancel{x+5}_{1}}$$

$$= x+2$$

Skill recap: Recap

$$x^2+7x+10$$

can be factorised into 2 brackets:

$$(x+2)(x+5)$$

Which we can then use to find common factors when simplifying the algebraic fraction.

Example 2:

Simplify $\frac{7x^2+9x+2}{3x^2-x-4}$

$$=\frac{(7x+2)(x+1)}{(3x-4)(x+1)}$$

$$=\frac{(7x+2)\cancel{(x+1)}^{1}}{(3x-4)\cancel{(x+1)}^{1}}$$

$$=\frac{7x+2}{3x-4}$$

Example 3:

Simplify $\frac{x^2-16}{x^2-x-20}$

Watch out! Difference of 2 squares $a^2-b^2=(a+b)(a-b)$

$$=\frac{(x+4)(x-4)}{(x-5)(x+4)}$$

$$=\frac{\cancel{(x+4)}(x-4)}{(x-5)\cancel{(x+4)}}$$

$$=\frac{x-4}{x-5}$$

<u>Online clips</u>

U103, U437, U294

4 operations with Algebraic Fractions

Component Knowledge

- Able to use all 4 operations with fractions
- Simplify an algebraic fraction
- Factorise a quadratic to simplify

Key Vocabulary

Algebraic Fractions	A fraction whose numerator and/or denominator are algebraic expressions
Factorise	Using Highest Common Factors (HCF) to simplify an expression
Simplify	Reducing the expression/fraction to a simpler form.
Numerator	The top number in a fraction. Shows how many parts we have
Denominator	The bottom number in a fraction. Shows how many equal parts the numerator is divided into.

We can add and subtract algebraic fractions like we do with numerical fractions.

- **Adding and Subtracting** = the key rule here is to find the **common denominator.**

Eg.1 Simplify

$$\frac{2x}{5} + \frac{x}{2}$$

Lowest common denominator is 10

x2 / x5

$$= \frac{2x}{5} + \frac{x}{2}$$

$$= \frac{4x}{10} + \frac{5x}{10}$$

$$= \frac{9x}{10}$$

Eg 2 Simplify

$$\frac{x-8}{3} - \frac{x-5}{7}$$

Lowest common denominator is 21

x7 / x3

$$= \frac{x-8}{3} - \frac{x-5}{7}$$

$$= \frac{7(x-8)}{21} - \frac{3(x-5)}{21}$$

$$= \frac{7(x-8) - 3(x-5)}{21}$$

$$= \frac{7x - 56 - 3x + 15}{21}$$

$$= \frac{4x-41}{21}$$

Eg 3 Simplify

$$\frac{2}{x+3} + \frac{1}{x}$$

Lowest common denominator is $x(x+3)$

x x / x $x+3$

$$= \frac{2}{x+3} + \frac{1}{x}$$

$$= \frac{2(x)}{x(x+3)} + \frac{1(x+3)}{x(x+3)}$$

$$= \frac{2x + x + 3}{x(x+3)}$$

$$= \frac{3x + 3}{x(x+3)}$$

Note: this fraction cannot be simplified further as there are no common factors, even though it looks like it should do.

We can divide or multiply algebraic fractions like we do with numeric fractions.

- **Multiplying and dividing =** the key rule here when multiplying is to multiply the numerators and the denominators.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Eg.1: $Calculate:$ $\frac{2x}{7} \times \frac{3y}{4}$	Eg 2: $Calculate:$ $\frac{4x}{3y} \times \frac{2y}{6x}$
$= \frac{2x \times 3y}{7 \times 4}$ $= \frac{6xy}{28}$ $= \frac{3xy}{14}$	$= \frac{4x \times 2y}{3y \times 6x}$ $= \frac{8xy}{18xy}$ (We can cancel common factors here) $= \frac{8xy}{18xy}$ (We can simplify the fraction here) $= \frac{8}{18} = \frac{4}{9}$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Eg 3: $Calculate:$ $\frac{2x+1}{2} \div \frac{4x+2}{4}$
$= \frac{2x+1}{2} \times \frac{4}{4x+2}$ (Multiply the first fraction by the reciprocal of the second) $= \frac{4(2x+1)}{2(4x+2)}$ $= \frac{8x+4}{8x+4}$ (We can simplify the fraction here) $= 1$

Online Clips

U685, U457, U824

Solving Algebraic Fractions

Component Knowledge

- Solve equations involving algebraic fractions.

Key Vocabulary

Solving	To find a value (or values) we can put in place of a variable that makes the equation true.
Equation	An equation states that two things are equal.

Problems are often answered in mathematics by solving equations.
To solve them, we need to find what number the variable in the equation represents. That number is called the solution of the equation.

Recap : Solving an Equation

(We use tramlines and work through the equation to make x the subject)

$$\frac{22+2x}{3} = 12 \quad (\times 3)$$

$$22 + 2x = 36 \quad (-22)$$

$$2x = 14 \quad (\div 2)$$

$$x = 7$$

Example: Solve: $\frac{x+3}{3} - \frac{x-4}{5} = 3$

$$\frac{x+3}{3} - \frac{x-4}{5} = 3$$

Step 1: Find a common denominator of the fractions.

$$\frac{5(x+3) - 3(x-4)}{15} = 3$$

Step 2: Simplify the numerators by collecting like terms.

$$\frac{5x+15-3x+12}{15} = 3$$

Step 3: Solve the equation.

x15 | x15

$$2x + 27 = 45$$

- 27 | - 27

$$2x = 18$$

÷2 | ÷2

$$x = 9$$

Sometimes to solve algebraic fractions, we need to first solve a quadratic by factorising.
Here is an example of an exam style question.

Solve $\frac{3}{x+5} + \frac{1}{x+3} = 2$

$$\frac{3(x+3) + x+5}{(x+5)(x+3)} = 2$$

Find the common denominator

Expand and Simplify

$$3x + 9 + x + 5 = 2(x^2 + 8x + 15)$$

Simplify by multiplying both sides by $(x + 5)(x + 3)$

$$4x + 14 = 2x^2 + 16x + 30$$

$$0 = 2x^2 + 12x + 16$$

$$0 = x^2 + 6x + 8$$

Factorise the quadratic to solve

$$0 = (x+4)(x+2)$$

$x = -4 \quad x = -2$

If you cannot solve by factorising, then you will need to use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Online Clips

U505, U150, U294

Probability with Algebra

Component Knowledge

- To be able to calculate the probability of events when one event is unknown.

Key Vocabulary

Probability	The extent to which something is likely to happen
Probability Tree	A visual representation of multiple events concerning probability
Event	A thing that happens or takes place
Conditional	When an event is affected by a previous event that has occurred
Unconditional	When an event is **not** affected by a previous event that has occurred
Expression	A collection of algebraic terms (does not contain an equals sign)
Equation	Something that states that 2 algebraic expressions are equal

Unconditional Probability

A fair spinner has ten equally likely sectors. n of the sectors are black and the rest are white.

Probabilities are the same for both spins.

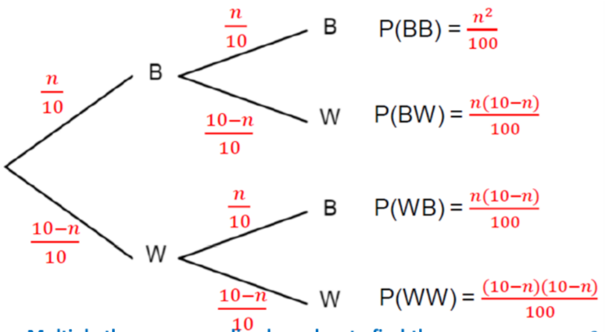
(a) The spinner is to be spun once.
(b) Write expressions in terms of n for:

(i) P(black) = $\frac{n}{10}$ (ii) P(white) = $\frac{10-n}{10}$

We do not know how many black sectors we but we know there are 10 total. So we can form expressions as shown above.

(b) The spinner is to be spun twice. Complete the tree diagram with expressions in terms of n for the probabilities.

We can then use the fractions to complete the probability tree.



Multiply the corresponding branches to find the probability of the 2 events occurring.

E.g.: $\frac{n}{10} \times \frac{n}{10} = \frac{n^2}{100}$

(c) Given that when the spinner is spun twice, the probability that it lands on black both times is $\frac{16}{25}$, work out the value of n.

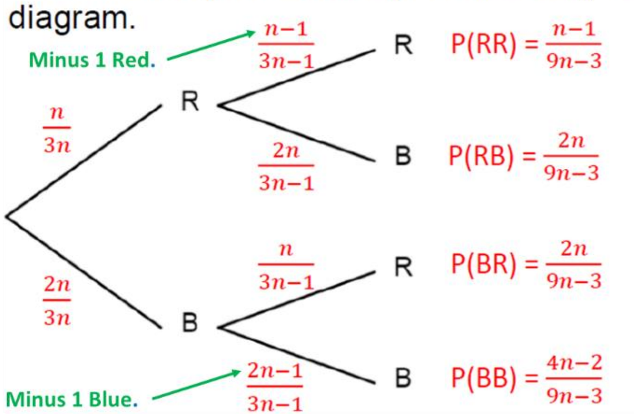
Form an equation and solve to find n. $\frac{n^2}{100} = \frac{16}{25}$ $n = 8$

Conditional Probability

A bag contains red and blue counters.
There are n red counters and twice as many blue counters.

In this example the counters are not replaced. This means that the probabilities on the second pick are different

(a) Two counters are picked at random without replacement.
Work out the probabilities (in terms of n) for the tree diagram.



Again, multiply the branches to find the probability of the 2 events occurring.

As the counters are not replaced this means that there is one less total so we must take 1 from the denominator.

The probability that the counters are both red is $\frac{1}{10}$.

(b) Work out the value of n.

$P(RR) = \frac{n-1}{9n-3}$

Therefore, we can form the equation.

$$\frac{n-1}{9n-3} = \frac{1}{10}$$

$$10n - 10 = 9n - 3$$

Solve the equation to find n.

$$n = 7$$

Online clips

U806, U457, U685

Algebraic Proof

Component Knowledge

- To be able to demonstrate or show that a statement is true by using examples or counterexamples.
- To be able to prove a statement is always true using an algebraic method.

Key Vocabulary

Topic/skill	Definition	Example
Expression	A mathematical statement written using symbols, numbers or letters.	$3x + 2$ or $5y^2$
Equation	A statement showing that two expressions are equal.	$2y - 17 = 15$
Identity	An equation that is true for all values of the variables. An identity uses the symbol ≡.	$2x \equiv x + x$
Coefficient	A number used to multiply a variable. It is the number that comes before / in front of a letter.	6z 6 is the coefficient, z is the variable
Odd and even	An even number is a multiple of 2. An odd number is an integer which is not a multiple of 2. (one more or less than an even number)	If n is an integer (whole number) An even number can be represented by 2n or 2m etc. An odd number could be represented by 2n -1, or 2n + 1, or 2m + 1 etc.
Consecutive Integers	Whole numbers that follow each other in order.	If n is an integer, n, n + 1, n + 2 etc are consecutive integers.
Sum	The sum of two or more numbers is the value you get when you add them together.	The sum of 4 and 6 is 10. The sum of n and n + 1 is 2n + 1.
Product	The product of two or more numbers is the value you get when you multiply them together.	The product of 4 and 6 is 24. The product of m and n is mn.
Multiple	To show that an expression is a multiple of a number, you need to show that you can factor out the number.	12m is a multiple of 4 because it can be written as 4 x 3m. m^2 is a multiple of m because it can be written as m x m.
Square Terms	A term that is produced by multiplying another term by itself.	If n is an integer n^2 are square integers.
Difference	Subtracting the smaller value from the larger value.	The difference between 2n + 1 and n is (2n + 1) – n = n + 1.

Some useful generalisations

Even numbers	2n, 2n+2, 2n-2, 2m, 2p …
Odd numbers	2n + 1, 2n-1, 2n+3, 2m+1, 2p+1 …
Consecutive integers	n, n+1, n+2, …
Consecutive even numbers	2n, 2n+2, 2n+4, …
Consecutive odd numbers	2n+1, 2n+3, ….
Even number + even number	= Even number
Even number + odd number	= Odd number
Odd number + even number	= Odd number
Odd number + Odd number	= Even number

Example 1

Prove that the square of an odd number is always odd.

Let the odd number be $2n + 1$.

So, $(2n+1)^2 = (2n+1)(2n+1)$

$= 4n^2 + 4n + 1$

$= 2(2n^2 + 2n) + 1$

We can factorise part of the expression, $4n^2 + 4n$, into $2(2n^2 + 2n)$, which is an even number as it is a multiple of 2.

As $2(2n^2 + 2n)$ is even and +1 is odd, Even number + Odd number = Odd number

Example 2

The sum of any three consecutive integers is a multiple of 3.

Let the consecutive integers be $n, n+1$ *and* $n+2$.

So $n + \ n+1 \ + \ n+2 \ = 3n + 3$

$= 3(n+1)$

This is a multiple of 3.

Example 3

Prove for any 3 consecutive integers the difference between the product of the first 2 and the product of last two is always twice the middle number.

Let the consecutive integers be $n, n+1$ *and* $n+2$.

The product of the first and second

$$n\,(n+1) = n^2 + n$$

The product of the second and third

$$(n+1)\,(n+2) = n^2 + 3n + 2$$

So the difference between these products is

$$n^2 + 3n + 2 \ - \ n^2 + n \ = 2n + 2$$

This equals $2\,(n+1)$ which is twice the middle number.

Example 4

Prove that **any two odd numbers** sum to an even number.

We use $2n + 1$ for the first number and $2m + 1$ for the second number.

We use two different letters, m and n, so that the odd numbers are not related.

We add the two odd numbers:

We can remove a factor of 2 from each term.

$$2n + 1 + 2m + 1 = 2n + 2m + 2 = 2(n + m + 2)$$

The expression $2(n + m + 2)$ is a multiple of 2, so **it is even.**

Online clip
U582

Trigonometric graphs

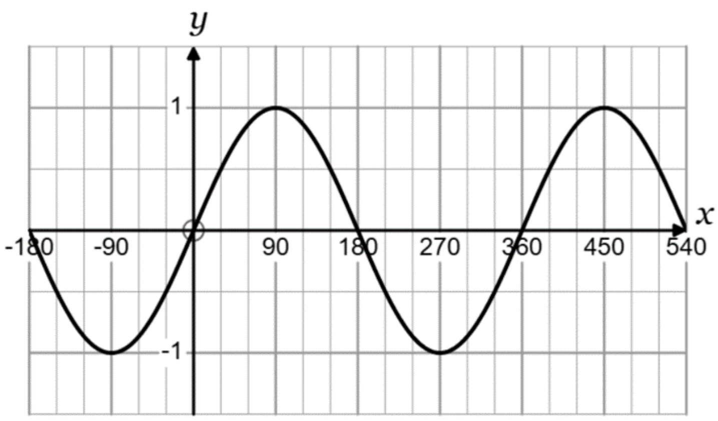
Component Knowledge

- Recognise the sine, cosine and tangent graphs
- Recognise symmetry in the sine, cosine and tangent graphs
- Use the sine, cosine and tangent graphs to solve equations

Key Vocabulary

Sine	The ratio of the opposite side given the angle to the hypotenuse
Cosine	The ratio of the length of the adjacent side to the given the angle to the hypotenuse
Tangent	The is the ratio of the side opposite the given the angle to the adjacent side
Periodic	A graph which repeats itself over and over at regular intervals.
Asymptote	A line which a graph gets closer and closer to but does not touch or cross a specific point or line on a graph.

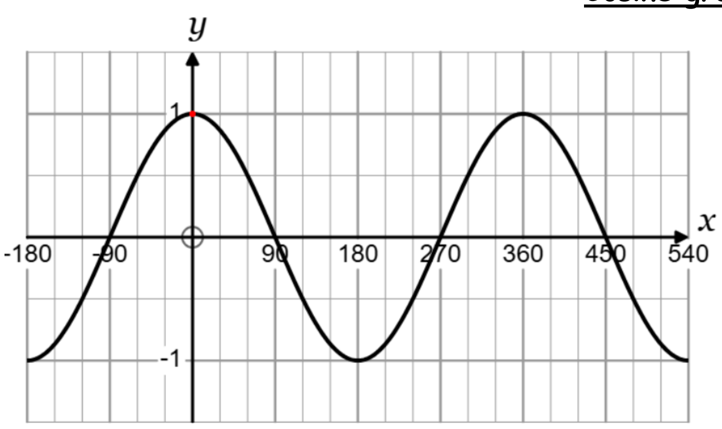
Sine graph



$y = sin\ x$

- Periodic every 360°
- The maximum value of sinθ is 1.
- The minimum value of sinθ is −1.
- Key coordinates:
  (0, 0)
  (90, 1)
  (180, 0)
  (270, - 1)
  (360, 0)

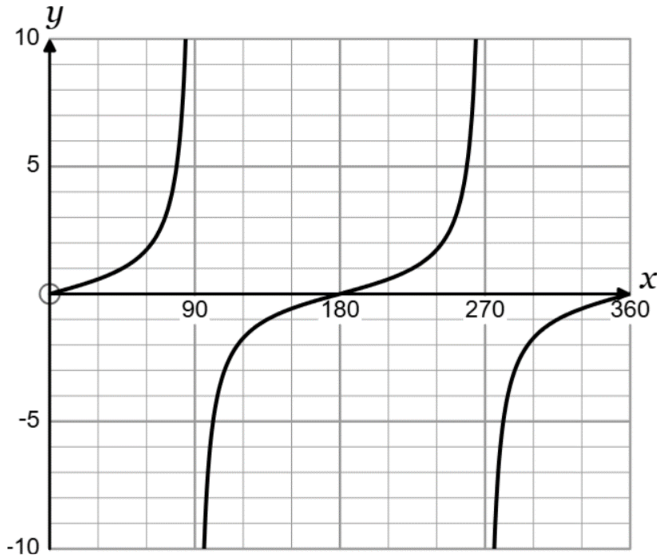
Cosine graph



$y = cos\ x$

- Periodic every 360°
- The maximum value of cosθ is 1.
- The minimum value of cosθ is −1.
- Key coordinates:
  (0, 1)
  (90, 0)
  (180, - 1)
  (270, 0)
  (360, 1)

Tangent graph



$y = tan\ x$

- Periodic every 180°
- Key coordinates:

  (0, 0)

  x = 90 is an asymptote

  (180, 0)

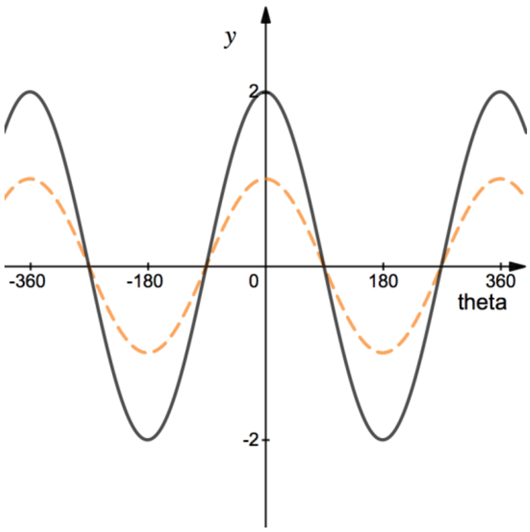
  x = 270 is an asymptote

  (360, 0)

Transforming trig graphs

A trigonometric graph can also be transformed. We may need to sketch them or use them to solve equations.

In the diagram below, the dashed line represents y = cosθ and the solid line represents y = 2cosθ (a stretch in the y axis).



Solve equations using trig graphs

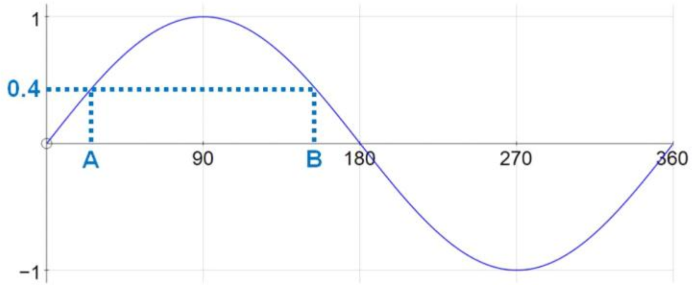
A trigonometric graph can be used to find solutions to a trigonometric problem. It can be particularly useful when there are multiple solution, which solving algebraically may miss.

Eg for $0° \leq x \leq 360°$, solve sin x = 0.4 to 1 d.p.

$A = \sin^{-1} 0.4 = 23.6°$ (1dp)

$B = 180 - A = 156.4°$ (1dp)

Solutions are 23.6° and 156.4°



Online clip

U450